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Validity Evidence for the Middle School Mathematical Beliefs Scales (MMBS)

Evidencias de validez para las Escalas de Creencias Matemáticas para Educación Media (MMBS)

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Abstract. *Objective*. Students' beliefs about themselves and about what it means to be effective learners of mathematics have been shown to influence multiple factors, such as their success, engagement, and motivation. The purpose of this paper is to report evidence of validity for middle school mathematical beliefs scales (MMBS). *Method*. Confirmatory Factor Analysis (CFA) was used to examine the reliability and construct validity of the MMBS, using data drawn from a sample of 248 middle school students from a single suburban school on the East Coast of the United States. *Results*. Findings suggest that the MMBS has a solid six-factor structure that demonstrates robust reliability and construct validity.

Keywords. Mathematics education, beliefs, attitudes, middle school education, validity evidence

Resumen. *Objetivo*. Se ha demostrado que las creencias de los estudiantes sobre sí mismos y sobre lo que significa ser aprendices eficaces de matemáticas influyen en múltiples factores, tales como su éxito, compromiso y motivación. El propósito de este artículo es presentar evidencias de validez para las escalas de creencias matemáticas en educación secundaria (MMBS, por sus siglas en inglés). *Método*. Se utilizó el Análisis Factorial Confirmatorio (AFC) para examinar la fiabilidad y la validez de constructo de las MMBS, empleando datos de una muestra de 248 estudiantes de secundaria de una escuela suburbana en la costa este de los Estados Unidos. *Resultados*. Los hallazgos sugieren que las MMBS poseen una sólida estructura de seis factores que demuestra una fiabilidad y una validez de constructo robustas.

Palabras clave. Educación matemática, creencias, actitudes, educación media, evidencias de validez

Introduction

It is well established that the beliefs students hold about the nature of learning, mathematics, and their own ability impact on their engagement, motivation, success, and even the strategies that they use (Boaler, 2016; House, 2006; Mazana et al., 2019). While definitions vary, beliefs are operationalized herein as subjective, value-laden mental constructs that individuals perceive as accurate (Philipp, 2007; Skott, 2014). Researchers have categorized mathematical beliefs into beliefs about the nature of mathematics, beliefs about learning mathematics, and beliefs about one's mathematical ability (Di Martino & Zan, 2015).

Although various scales have been created to individually measure each of these specific categories, few scales exist that measure them across multiple categories. Moreover, within existing scales, many include items that are framed in ways that are not asset-based and/or that may inadvertently cause students to consider unproductive beliefs. Thus, there is a need to develop and collect evidence of validity on asset-based mathematical belief scales. The present study was guided by the following research question: To what extent is there evidence of content, response-process, and internal structure validity to support the use of the MMBS when measuring mathematical beliefs in middle school students?

Defining mathematical beliefs

While definitions vary, beliefs are often seen as subjective, value-laden mental constructs that individuals perceive as accurate (Philipp, 2007; Skott, 2014). Beliefs around mathematics form interconnected clusters and are commonly categorized into beliefs concerning the nature of mathematics, its learning, its teaching, the relationship of oneself with the subject (Beswick, 2012; Boaler, 2016; Ernest, 1989). Though traditionally considered stable, research suggests that students' beliefs can evolve with experience (Philipp, 2007; Skott, 2014).

Influence of mathematical beliefs on students

Research on mathematics education has found critical impacts of students' mathematical beliefs on their learning processes, engagement, and achievements (House, 2006; Leder & Forgasz, 2002; Mazana et al., 2019). For example, beliefs related to the nature of mathematics can frame it either as a rigid, procedural discipline or as an inherently dynamic, exploratory activity, greatly influencing student's attitudes and approaches toward the subject (Schoenfeld, 2010).

Within beliefs about learning mathematics and learning in general, students who view mathematical ability as a typically innate experience showed increased frustration and anxiety during problem-solving tasks, often misinterpreting their struggles as an indicative of overall intelligence deficits (Boaler, 2016). These beliefs can also influence how students approach mathematical tasks. Some attribute success to natural ability, while others recognize effort, persistence, and practice as critical to their achievement (Hannula et al., 2019). According to Boaler (2016), students may adopt a growth mindset in general life scenarios but maintain a fixed one in academic subjects such as mathematics, believing their ability is innate and unchangeable. This fixed mindset may cause significant discouragement and distress when students struggle to solve mathematical problems, reinforcing a misconception that mathematical talent is indicative of overall intelligence. Similarly, a comparative study by House (2006) involving thousands of elementary students in Japan and the United States found that students believing in hard work and studying at home scored higher in mathematics, whereas those who struggled often considered the subject dull and dependent on innate ability. Although challenging, Blackwell et al. (2007) demonstrate that transitioning from a fixed mindset to a growth one can significantly enhance students' mathematical progress.

Beliefs regarding self and personal mathematical ability profoundly influence motivation and engagement. Mazana et al. (2019) highlight that students' perceptions on their abilities and learning process significantly affect their problem-solving strategies, persistence, and overall mathematical success. Addressing deeply held misconceptions and promoting accurate beliefs about mathematics and its learning are essential for educators and policymakers seeking to foster supportive educational environments that enhance mathematical engagement, persistence, and achievement (Mazana et al., 2019).

Factors of the MMBS

The MMBS was structured to have factors targeting beliefs in each of the categories of beliefs. Consequently, the literature review that follows further unpacks the definition of each of the six factors addressed in the MMBS: self-efficacy; growth mindset; valuing mistakes, struggle, and persistence; speed; using experiences in school mathematics; and mastery orientation. In addition, each section discusses the relationship between that factor and mathematics learning. Given its foundational role in shaping students' mathematical beliefs and behaviors, self-efficacy serves as an appropriate starting point for this review.

Self-efficacy

Self-efficacy was defined by Bandura (1977) as one's belief in their own capability to be successful in a situation or specific task. Given this context-specific definition, self-efficacy is intrinsically tied to the characteristics of the task, making it different from generic definitions of self-confidence (Pajares & Miller, 1994). Students with higher self-efficacy tend to demonstrate more resilient and self-regulated learning behaviors, such as using more effective strategies and persisting through challenges (Bandura, 1977; Schunk & Pajares, 2002; Zakariya, 2022). In mathematics, self-efficacy has been found related to better problem-solving skills, lower math anxiety, higher performance, and the increased use of strategies for self-regulation (Hoffman & Schraw, 2009; Usher & Pajares, 2009; Schunk & Pajares, 2002; Zimmerman, 2000).

Effective measurement of mathematics self-efficacy often employs task-specific instruments, such as the Mathematics Self-Efficacy Scale (MSES) developed by Betz and Hackett (1983) and the Self-Efficacy for Learning and Performance scale from the Motivated Strategies for Learning Questionnaire (Pintrich et al., 1991). These instruments frequently use Likert-scale questions targeted at specific mathematical tasks. Additionally, qualitative techniques, including think-aloud methods and interviews, provide deeper insights into students' self-efficacy beliefs and their impact on mathematical learning processes (Bandura, 2006).

High self-efficacy significantly enhances mathematics learning by promoting superior problem-solving skills, reducing math-related anxiety, and boosting overall academic performance (Hoffman & Schraw, 2009; Usher & Pajares, 2009; Schunk & Pajares, 2002; Zimmerman, 2000). Students with strong self-efficacy tend to willingly engage with challenging tasks, to exhibit resilience and persistence when confronted with difficulties, and to adopt effective learning strategies (Schunk & Pajares, 2002). Such students are also prompt to set clear academic goals and employ successful self-regulation practices, which considerably influence their academic outcomes (Zimmerman, 2000).

Growth mindset

The term growth mindset was coined by Carol Dweck (2006) after observing that children responded differently to challenges and setbacks. Specifically, she defined growth mindset as the belief that abilities and intelligence can grow over time. This contrasts with fixed mindsets or the belief that one's ability remains relatively stable. Those with a growth mindset are generally more resilient, motivated, and persistent in their learning, viewing setbacks as opportunities rather than as indicators of inadequacy (Yeager et al., 2019).

Research on growth mindset has been mixed with some studies suggesting that having a growth mindset is associated with greater academic achievement, resilience, willingness to take risks, problem-solving abilities, and a stronger focus on learning (Blackwell et al., 2007; Boaler, 2016; Burnette et al., 2013; Claro et al., 2016; Yeager & Dweck, 2020). Specifically, growth mindset beliefs have been linked to improved self-regulation in learning and reduced test anxiety (Burnette et al., 2013). Claro et al. (2016) found that students with growth mindsets showed greater perseverance when facing academic challenges and achieved better academic outcomes.

However, other research highlighted that the effectiveness of growth mindset interventions varies significantly based on environmental, social, and cultural factors (Sisk et al., 2018). A recent meta-analysis by Macnamara and Burgoyne (2023) found nonsignificant overall effects of growth mindset interventions when controlling factors like publication bias, suggesting that growth mindset interventions alone may not universally improve academic performance.

Growth mindset is commonly measured using various approaches, including behavioral tasks, self-report questionnaires, and intervention studies. One widely used self-report instrument is the Implicit Theories of IQ Scale (ITIS), developed by Dweck (1999). It assesses individuals' beliefs regarding intelligence on a continuum from fixed to growth perspectives. Additionally, researchers employ motivational assessments to investigate how students respond to setbacks, persist in problem-solving tasks, and approach challenging tasks (Blackwell et al., 2007).

Recent brain-imaging research has also explored variations in neural responses to feedback and errors, revealing differences among individuals holding distinct mindset orientations (Moser et al., 2011). However, self-report measures face criticism for their limited ability to accurately capture authentic mindset beliefs, leading researchers to develop more objective, behaviorally based assessments (Fischer et al., 2019).

Despite mixed findings, the growth mindset remains influential in educational research and is widely recognized as essential for promoting adaptive learning behaviors and resilience in educational settings (Yeager & Dweck, 2020). This emphasis on adaptive learning behaviors naturally leads to the significance of valuing mistakes, struggle, and persistence in learning processes.

Valuing mistakes, struggle, and perseverance

Valuing mistakes, challenges, and perseverance in learning demonstrates that encounter difficulties is vital in learning processes (Ingram et al., 2013). Research has indicated that committing errors and facing challenges lead to a deeper cognitive engagement, allowing students to improve their understanding of concepts (Kapur & Bielaczyc, 2012). In mathematics education, valuing mistakes, difficulties, and persistence is related to students' believes that facing errors and challenges is essential for learning—particularly when solving challenging mathematical problems—. Besides, it is linked to resilience (Duckworth et al., 2007) and constructive failure (Kapur, 2008). Additionally, students can strengthen their understanding and improve cognitive processing by engaging with their mistakes and challenges (Metcalfe, 2017). Moreover, those who perceive mistakes as chances to learn rather than failures develop greater resilience and problem-solving skills (Hattie & Timperley, 2007).

Furthermore, research demonstrates that students who embrace mistakes and persist through struggles exhibit greater motivation, self-regulation, and overall academic performance (Dweck, 2006; Yeager & Dweck, 2020). Teachers play a critical role in promoting constructive struggle, thereby strengthening students' problem-solving skills and resilience, essential traits for lifelong learning (Rattan et al., 2012). However, providing adequate support during struggles is vital, as insufficient assistance can lead to student frustration and disengagement (Hiebert & Grouws, 2007).

Evaluating students' attitudes toward mistakes, challenges, and perseverance commonly involves behavioral tasks, self-report surveys, and neuroscientific methods. Instruments such as the Achievement Goal Questionnaire (AGQ; Elliot & Murayama, 2008), the Grit Scale (Duckworth & Quinn, 2009), and the Math Anxiety and Attitudes Scale (MAAS; Ramirez et al., 2013) measure students' emotional responses and resilience in mathematics. Observational methods have also tracked persistence through challenging tasks and problem-solving strategies, while neuroscientific studies have provided insights into brain activities related to error processing and adaptability (Kaplan & Patrick, 2016; Moser et al., 2011). Understanding the importance of valuing mistakes, struggle, and persistence lays a strong foundation for examining how beliefs about speed and fluency further impact mathematics learning.

Speed

Speed in mathematics refers specifically to how quickly students can process and resolve problems, encompassing both accuracy and efficiency (Ashcraft, 1992). A key tension in mathematics education concerns whether excellence in mathematics is related to attributes typically associated with procedural fluency, such as speed and accuracy, or to a more holistic conception to mathematics proficiency that incorporates other key strands, such as conceptual understanding, adaptive reasoning, strategic competence, and productive dispositions (Findell et al., 2001).

Performing mathematical computations and solving problems quickly is frequently emphasized through classroom practices such as fluency drills, timed assessments, and standardized testing (Geary, 2011). In contrast, increased fluency has been shown to correlate with academic performance (Cowan et al., 2006). However, an excessive focus on speed can also have detrimental outcomes for the learning process as students may prioritize quick answers over accuracy and conceptual comprehension, which could exacerbate math anxiety (Ramirez et al., 2013).

Research has reinforced the necessity of balancing speed and accuracy with conceptual understanding to achieve long-term educational success and advocate for learning environments that accommodate varying processing speeds, reducing pressure and promoting deeper cognitive engagement (Boaler, 2015; Tolar et al., 2012). Students who perceive speed as an indicative of mathematical proficiency often equate faster problem-solving with higher ability, leading to increased anxiety among those who struggle in speed-focused environments (Beilock & Maloney, 2015). Conversely, students valuing conceptual understanding typically engage more deeply and display greater resilience when encountering challenging mathematical problems (Khoule et al., 2017). Thus, some researchers have argued that education should prioritize conceptual understanding over rapid processing, advocating for reduced reliance on timed assessments to support deeper learning (Boaler, 2016).

Speed is typically assessed through timed tests, fluency evaluations, and cognitive processing tasks such as the Woodcock-Johnson III Tests of Achievement or the rapid naming tasks (Woodcock et al., 2001; Denckla & Rudel, 1976). Neuroimaging and eye-tracking studies have further provided insights into cognitive load and problem-solving efficiency related to speed (Grabner et al., 2007). Timed assessments emphasize rapid retrieval and procedural execution, capabilities often associated with automaticity—the ability to perform tasks with minimal cognitive effort (Logan, 1988). However, quick recall does not necessarily indicate deep conceptual understanding, which emphasizes that true mastery in mathematics involves more than speed alone (Rittle-Johnson & Schneider, 2015).

Thus, student beliefs about whether speed is more important than understanding in learning mathematics often influences their approach to learning mathematics and long-term success. Despite this, few prior studies have sought to measure student beliefs in this domain.

Using experiences in school mathematics

Understanding the role and implications of speed in mathematics education sets the stage for exploring a broader context of students' school mathematics experiences. Research has suggested that supporting students in seeing how the mathematics learned in schools connects to their lives is essential in developing positive dispositions toward the subject (Benson-O'Connor et al., 2019) and increasing their motivation (Fennema et al., 1981). Many existing scales focus on students' beliefs around the usefulness of mathematics—the degree to which mathematics learned in school is applicable to their real lives (Kloosterman & Stage, 1992). However, given the importance of building mathematical knowledge drawing on students' funds of knowledge (Land et al., 2019), the present study operationalize this category as both the belief that school mathematics is useful in the real world as well as the belief that daily experiences can help make sense of mathematical concepts.

Mastery orientation

Mastery orientation refers to an individual's intrinsic motivation to develop competence and master new skills rather than focusing on external validation or outperforming others (Dweck & Leggett, 1988). This contrasts with performance orientation, where individuals focus on demonstrating their abilities to gain approval or avoid negative judgments (Elliot & Dweck, 1988). Specifically, individuals attach a positive interest and invest genuine efforts in a task to master or increase their competence while being intrinsically motivated (Ames & Archer, 1988; Furner & Gonzalez-DeHass, 2011).

Students with a mastery orientation typically perceive challenges as opportunities for growth and learning, taking pride in gaining new knowledge and maintaining persistence despite encountering setbacks (Martin et al., 2008). In this way, mastery orientation aligns closely with growth mindset, reinforcing the belief that intelligence and skills can be developed through sustained effort and perseverance (Dweck, 2006; Grant & Dweck, 2003).

Research suggests that mastery orientation is associated with numerous positive outcomes, such as greater intrinsic motivation, deeper cognitive engagement, academic efficacy, higher resilience when facing academic challenges, and increased tendencies to use adaptive learning strategies, such as seeking help when necessary and employing effective problem-solving techniques (Ames & Archer, 1988; Pintrich, 2000; Furner & Gonzalez-DeHass, 2011). This contrasts with students with performance orientations who may experience increased anxiety, avoidance behaviors, and a fear of failure, which can hinder long-term academic success (Midgley et al., 2001). However, its impacts are also dependent on classroom contexts structured around mastery goals, where teachers emphasize learning and personal improvement as central measures of success (Martin et al., 2008).

The purpose of the present study was to develop and collect initial evidence of validity—including evidence of the psychometric properties of the scales—supporting the use of the scales to measure middle school students' mathematical beliefs. These scales are intended to measure the mathematical beliefs of middle school students and to explore the relationship between these beliefs and other factors (e.g., problem-solving proficiency). The scales should not be used to group students or predict future academic achievement

Method

Participants

The present study was part of a larger study exploring mathematical problem solving among middle school students that was approved by the Advarra IRB under protocol Pro00067349. Further, this study employed a cross-sectional, non-probabilistic (correlational) design to validate the instrument. From a convenience sample, one school agreed to participate in the study. All mathematics teachers at the school were invited to participate. In accordance with the approved protocol, letters were sent to all students enrolled in courses taught by the participating mathematics teachers, as well as to their parents or guardians. Students and their parents or guardians had the option to opt out of the study. Students whose opt-out forms were not completed were also enrolled in the study. This yielded a total potential sample of 297 students. Within the present study, 248 students answered all items on the measure and were, therefore, included in the analyses. All participants were middle school students drawn from a single middle school located in a large, suburban district on the East Coast of the United States. The participants self-identified as girl (114), boy (130), non-binary (3), and 1 who did not self-report gender. There were 132 students enrolled in 7th grade and 116 in 8th grade.

Instruments

Para obtener los datos demográficos, se incluyeronTo develop the scales, evidence of content validity of both reviews of existing scales, frameworks, and research on mathematical beliefs (e.g., Dweck, 2006; Fennema & Sherman, 1976; Kirmizi & Tarim, 2022; Kloosterman & Stage, 1992; Usher & Pajares, 2009; Vazquez et al., 2020; Watson et al., 2021) were collected. From this, 34 original items were theorized to fall under the categories of growth mindset (8 items), self-efficacy (5 items), the nature of mathematics (15 items), and struggle and persistence (6 items). Additional evidence of content and response-process validity was then collected by having a panel of experts in mathematics and equity review the items. The goal of this review was to help ensure that the items were accessible to students, that they did not inadvertently convey negative messages about mathematics and of mathematics learning, and that they aligned to the intended constructs. After items were revised or removed based on feedback from expert reviewers, 29 from the original 34 remained.

These items then underwent two rounds of administration with students. Although a full discussion of this the first administration process is beyond the scope of the present manuscript, it is relevant to note that the 29 original items retained after expert reviewer were administered to a sample of 7th and 8th grade students, with 239 students having complete data across all items. An Exploratory Factor Analysis (EFA) with a common factor extraction (principal axis factoring) with an oblique rotation (direct oblimin) was then employed on the remaining 29 items. Items that did not load to theorized factors were examined and revised. In addition, new items and factors were added resulting in the 35-item eight-factor solution that was the starting point for the present study. Finally, evidence of response process validity was also collected through reading level analyses. These analyses revealed that the Flesh reading ease for all 35 items was 78.7 with a Flesh-Kincaid grade level of 5.2. This means that the reading level of the items was suitable for students in 5th grade or higher, and thus appropriate for the target population.

The resulting eight-factor scales, hereafter referred to as the Middle School Mathematical Beliefs Scales (MMBS), was administered to study participants via a Qualtrics survey. The items from the original and validated Mathematics Problem Solving Belief Scale developed by Kloosterman and Stage (1992) were adapted to fit middle school adolescents. Specifically, researchers entered each classroom to administer all study measures, although only the Qualtrics survey was relevant to the present study. For the survey, each student was given a laptop computer and asked to complete all items in the survey. Each item was given as a slide scale where respondents could select any value between 0 and 100, inclusive. The decision to use a continuous scale was based on research suggesting that these scales have greater reliability (Schraw, 2009) and on the desire to allow the use of MMBS scales in correlational analyses, which assume continuous data.

The directions of the survey asked the respondents to read carefully and select their answers by using the sliders placed next to each question. The closer they moved it to “0”, the LESS did they agree with sentence. On the contrary, the closer they moved it to “100”, the MORE did they agree with it.

In the case they left the slider in the middle, the answer would be that they NEITHER AGREE NOR DISAGREE with the statement. Participants were asked to answer as honestly as possible.

The MMBS that was administered consisted of latent variables of: (1) mathematics self-efficacy (6 items); (2) mathematics growth mindset (5 items); (3) valuing mistakes in math (5 items); (4) valuing struggle in math (3 items); (5) valuing persistence in math (4 items); (6) speed in performing math problems (5 items, including 1 reverse-coded); (7) beliefs about whether students' real-world experiences help them with mathematics (4 items); and (8) math mastery orientation (3 items). Moreover, reliability coefficients, McDonald's Ω, ranged from .77-.88.

Data analysis

The EQS 6.4 software (Bentler, 2005) was employed to conduct the series of standard CFA nested models described previously. Data was tested against requisite statistical assumptions and screened for outliers prior to data analysis. The data did not contain any univariate or multivariate outliers, and it met all requisite statistical assumptions except multivariate normality. No cases were deemed univariate or multivariate outliers; thus, all 248 cases were retained for analysis. Nevertheless, data demonstrated multivariate kurtosis (Mardia's Normalized estimate was 64.34; according to Bentler [2005] any values exceeding 6 are considered multivariate non-normal, with greater distance from 6 indicating greater degrees of multivariate non-normality). Thus, Maximum Likelihood Robust (MLR) statistics were requested in lieu of the normal distribution statistics. MLR statistics adjust standard errors of parameter estimates to account for the magnitude of multivariate non-normality; that is, a greater adjustment to standard errors is made for more severe violations of multivariate normality. Other assumptions such as homoscedasticity (i. e., box plots and scatterplots suggested a homoscedastic distribution among variables) and sphericity (i. e., Mauchley's Test of Sphericity, p = .45) were met.

The analysis began with a standard eight-factor CFA solution previously described, followed by the more parsimonious six-factor modified solution. The models were compared using the Satorra-Bentler scaled chi-square difference test (S-B $\Delta\chi^2$) for overall best fit to the observed data. Because MLR statistics correct for multivariate non-normality, comparing the models using a non-scaled normal distribution $\Delta\chi^2$ is inappropriate (Satorra & Bentler, 2010). Presumably, the model with the highest fit indices and lowest residuals would be a statistically significant improvement over all other models. Kline (2011) posits that "a special strength of SEM is that it offers much flexibility for modeling error variances and covariances in repeated measures designs" (p. 23). Hence, it treats auto-correlation in dependent data structures. Auto-correlation in the data was accounted by correlating the residuals of relevant manifest and latent variables. This was done, as recommended by Kline, for dependent data and to account for within-person shared variance. Pearson's r coefficients for the residuals ranged from r = .26 to r = .49, suggesting no multicollinearity among the residuals.

Goodness-of-fit indices (*NNFI, *CFI, *IFI) ≥ .90 suggest an adequately fitting model, and those ≥ .95 suggest excellent fit of the model to observed data. With respect to residuals, standardized root mean square residual (SRMR) values ≤ .11 suggest reasonable errors in estimating model parameters. For its part, root mean square error of approximation (*RMSEA) values ≤ .08 suggest that the model parameters approximate those of the population reasonably well (Byrne, 2006; Kline, 2011). Dillon-Goldstein's rho (ρ) was also used to assess the overall or composite reliability of the model. Rho measures how well the manifest or indicator variables, as a block, represent the latent variable in which they are hypothesized to load. Similarly to the interpretation of Cronbach's alpha, higher values for rho indicate greater model reliability, with .70 serving as the lower-bound for adequate model reliability (Werts et al., 1974).

Results

Eight-Factor Initial Solution

The eight-factor solution was ill-fitting to the observed data, S-B χ^2 (N = 248, df = 783) = 1215.11, p < .001, *NNFI = .883, *CFI = .893; *IFI = .895, SRMR = .084, RMSEA = .047 ($CI_{90\%}$ = .042, .052). Inspection of the standardized solution indicated that one item, the reverse-coded item in the beliefs about the importance of speed in performing math problems scale, exhibited a negligible factor loading; hence, the factor contributed negligible variance to this item. In addition, the three latent variables dealing with valuing (mistakes, struggles, persistence in math) demonstrated high multicollinearity (Pearson's $r \geq .90$). Finally, a review of the Lagrange Multiplier Test for adding parameters suggested that it was necessary to incorporate five error correlations among the indicators of the math growth-mindset and valuing latent variables. It also suggested that one cross-loading item (GM3), besides loading onto its original math growth-mindset latent variable, should be allowed to cross-load onto the math self-efficacy latent variable. Given that these modifications were theoretically defensible, this initial model was adjusted by making all these modifications to a respecified model. Figure 1 displays the configuration for this solution, and Table 1 displays the standardized factor loadings for each latent factor.

Figure 1. Eight-factor solution with latent variables, manifest variables, error terms, and inter-factor correlations

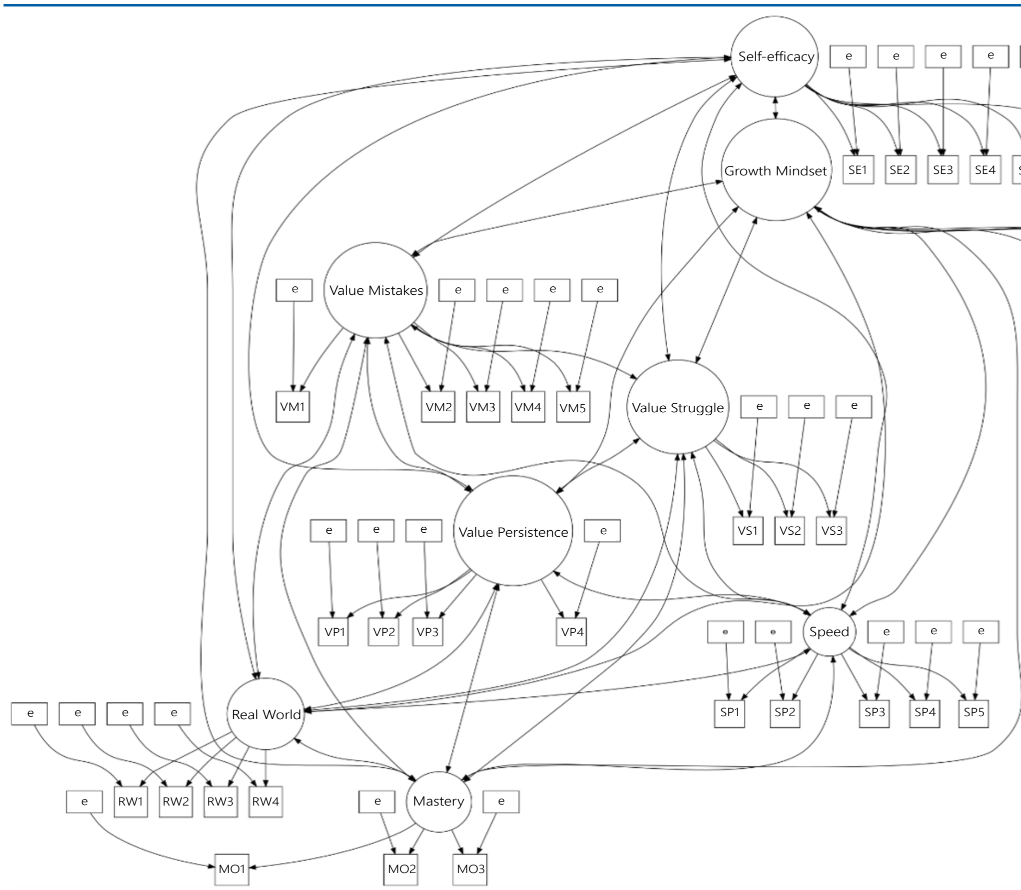

Table 1. Standardized Factor Loadings for the Eight-Factor Solution

Indicator	SE	GM	VM	VS	VP	SP	RWE	MO
VM2			.74					
VM3			.69					
VM4			.55					
VM5			.40					
VS1				.68				
VS2				.52				
VS3				.39				
VP1					.72			
VP2					.54			
VP3					.48			
VP4					.43			
SP1						.65		
SP2						.58		
SP3						.55		
SP4						.50		
SP5 (rc)						.23[NV]		
RWE1							.60	
RWE2							.49	
RWE3							.46	
RWE4							.38	
MO1								.66
MO2								.61
MO3								.49

Note. Key: SE = Math Self-Efficacy; GM = Math Growth Mindset; VM = Valuing Mistakes in Math; VS = Valuing Struggle in Math; VP = Valuing Persistence in Math; SP = Speed in Performing Math Problems; RWE = Beliefs about whether Students' Real-world Experiences Help Them with Math; MO = Math Mastery Orientation; (rc) = Reverse-Coded Item; [NV] = Negligible Variance.

Six-Factor Modified Solution

This model specified a modified six-factor structure compared to the eight-factor model, in which valuing mistakes in math, valuing struggle in math, and valuing persistence in math collapsed into one single factor rather than three separate ones. The modified six-factor solution fit the observed data reasonably well, S-B χ^2 (N = 248, df = 506) = 691.98, p < .001, *NNFI = .940, *CFI = .946; *IFI = .947, SRMR = .063, RMSEA = .039 ($CI_{90\%}$ = .031, .045). Standardized factor loadings were all significant and ranged from .332 to .886. Factor correlations were all positive and statistically significant, ranging from r = .28 to r = .79. The cross-loading item, GM3, exhibited a standardized factor loading of .598 in its original math growth-mindset latent variable and .372 in the math self-efficacy latent variable. Correlations between the error variances of the indicators previously mentioned were all positive and significant, ranging from r = .31 to r = 49. Finally, Dillon-Goldstein's rho = .958 suggested excellent composite model reliability. No other modifications made substantive theoretical sense; hence, this was deemed the final model. Figure 2 shows the configuration for this solution, and Table 2 presents the standardized factor loadings for each latent factor.

Figure 2. Six-factor solution with latent variables, manifest variables, error terms, and inter-factor correlations

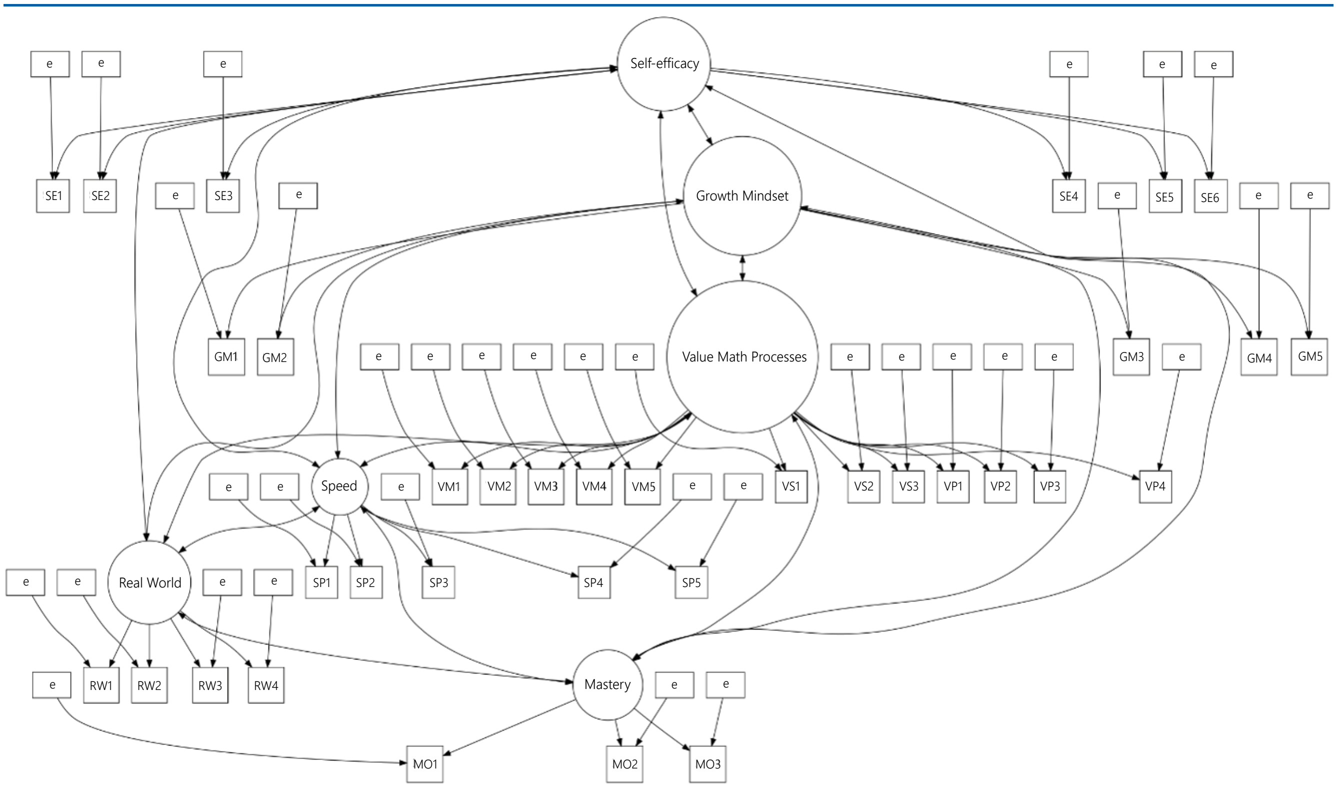

Table 2. Standardized Factor Loadings for the Six-Factor Solution

Indicator	SE	GM	MVB	SP	RWE	MO
SE1	.71					
SE2	.66					
SE3	.61					
SE4	.58					
SE5	.57					
SE6	.55					
GM1		.74				
GM2		.63				
GM3	.37	.59				
GM4		.56				
GM5		.51				
MVB1			.88			
MVB2			.86			
MVB3			.86			
MVB4			.82			
MVB5			.80			
MVB6			.74			
MVB7			.71			
MVB8			.69			
MVB9			.65			
MVB10			.62			
MVB11			.60			
MVB12			.51			
SP1				.65		
SP2				.59		
SP3				.52		
SP4				.46		
SP5 (rc)				.33		
RWE1					.71	

Continue

Indicator	SE	GM	MVB	SP	RWE	MO
RWE2					.68	
RWE3					.60	
RWE4					.52	
MO1						.78
MO2						.72
MO3						.65

Note. Key: SE = Math Self-Efficacy; GM = Math Growth Mindset; MVB = Math Beliefs about its Value; SP = Speed in Performing Math Problems; RWE = Beliefs about whether Students' Real-world Experiences Help Them with Math; MO = Math Mastery Orientation; (rc) = Reverse-Coded Item.

Table 3. Scaling Correction Factor Values of Nested Models

Model	S-B χ^2	*df*	SCF
Eight-factor	1215.11	783	.682
Six-factor	691.98	506	.658

Note. SCF = Scaling correction factor.

Table 4. Satorra-Bentler Scaled χ^2 Difference Test Results Between Nested Models

Model	CD*	Δdf	TRd**
Eight-factor, Six-factor	0.726	277	514.300***

Note. The model on the left is the comparison model and the model on the right is the alternative model.

* Difference in test scaling correction

** S-B scaled $\Delta\chi^2$ test statistic (T; TRd represents the mathematical equation used to calculate T)

*** S-B scaled $\Delta\chi^2$ test is significant at $p < .001$.

Comparison of Nested Models Employing the S-B $\Delta\chi^2$

Table 3 displays the scaling correction factor for the nested models, while Table 4 contains the results of the S-B $\Delta\chi^2$. As is evident from Table 2, the S-B $\Delta\chi^2$ is statistically significant ($p < .001$), which indicates that the six-factor model provides a superior statistical +fit to the observed data when compared to the eight-factor model. Hence, evidence suggests that more parsimonious six-factor solution should be retained in lieu of the more complex eight-factor solution.

Discussion

The present study sought to evaluate the validity of the Middle School Mathematical Beliefs Scales (MMBS) by examining evidence of content, response process, and internal structure. Beyond establishing acceptable model fit and reliability, the findings offer important insights into how middle school students organize and experience mathematical beliefs. In particular, the transition from the theorized eight-factor model to a more parsimonious six-factor structure provides a meaningful opportunity to examine the alignment between theoretical distinctions in the literature and the empirical structure of students' beliefs.

A central finding of this study is that the six-factor model provided a significantly better fit to the data than the original eight-factor model. Most notably, the constructs of valuing mistakes, valuing struggle, and valuing persistence converged into a single latent factor. While these constructs are often treated as theoretically distinct in prior research (e. g., Kapur, 2008; Duckworth et al., 2007), their empirical unification suggests that middle school students may not differentiate among these dimensions in practice. Instead, these beliefs may function as a cohesive orientation toward what has been described in the literature as productive struggle.

This finding aligns with theoretical perspectives suggesting that learners develop interconnected belief systems rather than isolated constructs (Philipp, 2007; Skott, 2014). From this perspective, students who view mistakes as valuable may simultaneously view struggle as necessary and persistence as beneficial, reflecting a broader epistemic stance toward learning mathematics. Such an interpretation is also consistent with research on growth-oriented learning environments, where errors, effort, and perseverance are framed collectively as integral to learning (Boaler, 2016; Kapur & Bielaczyc, 2012). Thus, the present findings extend prior work by suggesting that, at least for middle school students, these constructs may be more appropriately conceptualized and measured as a unified belief system rather than as discrete dimensions.

A second notable finding is the cross-loading of one growth mindset item onto the self-efficacy factor. This result suggests that students' beliefs about the malleability of their mathematical ability may not be fully distinguishable from their beliefs about their current competence. While growth mindset and self-efficacy are theoretically distinct constructs, one focused on beliefs about change and the other on beliefs about capability (Bandura, 1977; Dweck, 2006), the present findings indicate that these distinctions may blur in practice, particularly among younger learners.

This overlap may reflect developmental considerations, whereby students have not yet fully differentiated between "I can improve" and "I am capable." This result is consistent with research suggesting that motivational constructs often interact dynamically rather than functioning independently (Burnette et al., 2013; Yeager & Dweck, 2020). From a measurement perspective, this finding raises important questions about the discriminant validity of these constructs in middle school populations and suggests that future research should further investigate the conditions under which these beliefs can be empirically separated.

The findings related to the speed factor also warrant careful consideration. The relatively weak loading of the reverse-coded item suggests potential challenges in how students interpret items that contrast speed with conceptual understanding. This may reflect both methodological and conceptual issues. Methodologically, reverse-coded items are known to introduce cognitive complexity and measurement error, particularly among younger respondents. Conceptually, the tension between speed and understanding in mathematics may not be experienced by students as a clear dichotomy.

Prior research has emphasized that, while fluency and efficiency are associated with performance (Cowan et al., 2006), an overemphasis on speed can undermine conceptual understanding and increase anxiety (Beilock & Maloney, 2015; Ramirez et al., 2013). The present findings suggest that students' beliefs about speed may be more nuanced than commonly assumed and that measuring these beliefs requires careful attention to item phrasing and construct clarity. Future iterations of the MMBS may benefit from refining this factor to better capture the complexity of students' beliefs about fluency and understanding.

Taken together, these findings contribute to ongoing discussions regarding the structure and measurement of mathematical beliefs. Rather than simply confirming a predefined theoretical model, the results highlight areas of convergence and overlap among constructs that are often treated as distinct in the literature. This suggests that theoretical models of mathematical beliefs may benefit from greater attention to how these beliefs are organized and experienced by learners, particularly during early adolescence.

From a measurement perspective, the findings support the use of the MMBS as a multidimensional instrument with strong internal structure while also pointing to areas for refinement. The emergence of a unified factor related to productive struggle, the overlap between growth mindset and self-efficacy, and the challenges associated with measuring beliefs about speed all underscore the importance of iterative scale development grounded in both theory and empirical evidence. In this sense, the MMBS not only provides a tool for assessing mathematical beliefs but also serves as a mechanism for refining theoretical understandings of those beliefs.

The findings also have implications for educational practice. If students experience beliefs about mistakes, struggle, and persistence as a unified construct, instructional approaches that promote productive struggle may simultaneously influence multiple dimensions of students' beliefs. Similarly, the overlap between growth mindset and self-efficacy suggests that interventions targeting one construct may have spillover effects on the other. Educators may therefore benefit from designing learning environments that holistically support students' beliefs about effort, capability, and learning processes rather than addressing these constructs in isolation.

While these initial results are encouraging, it is important to acknowledge certain limitations. The current study was conducted with middle school students from a single suburban middle school. Therefore, the generalizability of these findings is limited. Future research should prioritize collecting additional evidence of validity on this instrument with more diverse groups of students, including those varying in age, greater range in socioeconomic status and geographical location, and differing mathematical background. Such cross-validation is essential to establish the instrument robustness and applicability across different populations. This will help determine whether the instrument psychometric properties, including its reliability and validity, hold across different contexts and student demographics.

Further research should also explore the relationship between the scores obtained using this instrument and other relevant constructs, such as student achievement in mathematics, engagement in mathematical tasks, and teacher perceptions of student mathematical abilities. Examining these relationships will provide a more comprehensive understanding of the instrument construct validity and its potential utility in educational settings. Longitudinal studies investigating how mathematical beliefs might evolve over time (using this measure), as well as more robust understandings of any potential consequences of utilizing the instrument in testing, would also be valuable. Within this, future research should explore whether the present scales can be reduced in length, thereby minimizing survey fatigue and the impact on instructional time within schools.

Another limitation of the study is that some of the factors (e. g., valuing struggle persistence, and mistakes) contain a significant number of items, which increases the risks of testing fatigue. Future research should consider if the scales can be shortened and balanced while retaining strong psychometric properties.

Finally, the present scale utilizes a trait-based approach to measuring student beliefs. Given that research has shown that beliefs are often impacted by state-based considerations (Middleton et al., 2017), future investigations should consider developing additional measures of state-based mathematical beliefs. By addressing these limitations and expanding the scope of study, a more nuanced understanding of the role of mathematical beliefs in student learning can be gained, and with it more effective interventions to support students' mathematical development.

The present study provides initial evidence supporting the validity of the MMBS while also offering important insights into the structure of middle school students' mathematical beliefs. By engaging directly with the results of the confirmatory factor analyses, the findings move beyond simple validation to inform theoretical and measurement considerations within the field. Future research should continue to examine the stability of this factor structure across diverse populations and explore how these beliefs relate to students' learning experiences and outcomes over time.

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Appendix

Final Six-Factor Model

Self-efficacy

1. I am confident that I can solve most math problems.
2. I am confident that I can solve math problems even when I don't know the answer right away.
3. I feel confident when solving challenging math problems.
4. I feel I can do well on math assignments.
5. When I walk into math class, I feel confident I will be able to understand the math.
6. I am confident that I can learn math that feels challenging.

Growth Mindset

1. I get better at math with practice.
2. The more I practice the better I get at math.
3. My math skills are constantly growing.
4. When a teacher goes over a math problem, I pay attention to how to solve it.
5. With practice, I believe I can get better at solving math problems.

Valuing Mistakes, Struggle, and Persistence

1. I learn from mistakes that I make in math.
2. When I make mistakes it can help me understand math better.
3. When I make mistakes while doing math, it helps me learn.
4. I don't mind making mistakes in math.
5. Making mistakes in math can be good for me.
6. When I take on a problem that is challenging, I become a better problem solver.
7. It is okay to struggle while solving math problems because it means that I am learning.
8. Struggling while solving math problems can help me get better at math.
9. I keep trying to solve a problem until I am successful.
10. I stick with a problem/keep trying to solve a problem even if I am having a hard time figuring out how to solve it.
11. Not giving up when I get stuck will help me get better at math.
12. I try different strategies in math to overcome feeling stuck.

Speed

1. It's good to slow down when solving math problems.
2. It's more important to think through a math problem than solve it quickly.
3. It's important to take my time when solving math problems.
4. How fast I solve a problem is less important than understanding a math problem.

Using Experiences in School Mathematics

1. My day-to-day experiences help me understand math.
2. The math I learn in school helps me understand the world around me.
3. I can use my day-to-day experiences to make sense of math problems.
4. I can use my experiences in life to help me learn math.

Mastery Orientation

1. The way I think about solving a math problem is just as important as the answer.
2. Being able to explain how I solved a math problem is just as important as getting the answer correct.
3. Learning is just as important as getting good grades.