Ciencia y Tecnología, 38(1): 28-33, 2022
ISSN: 2215-5708

EFFECTS OF TEMPERATURE IN ALPHA DECAY

J.E. Pérez Velásquez[1], N. G. Kelkar[2], O. L. Caballero[3]

[1,2] Departamento de Física, Universidad de los Andes, Cra. 1E N0. 18A-10, Bogotá, Colombia
[3] Department of Physics, University of Guelph, Guelph, ON N1G 2W1, Canada

Received May 2021; accepted January 2022

Abstract

One of the processes relevant in a nucleosynthesis calculation is the alpha decay (α-decay) of heavy nuclei. In this work, the alpha decay process is investigated in an astrophysical environment by introducing temperature dependence in the calculation of half-lives. By using a simple model based on the semiclassical treatment of the alpha decay process, the half-lives are evaluated by considering an alpha particle, which tunnels through the Coulomb barrier generated by its interaction with the daughter nucleus. We find the temperature dependent half-lives for several isotopes and compare them with the results obtained in a previous work. It is concluded that including temperature leads to a reduction in alpha decay half-lives. However, the magnitude of the reduction depends on the approach used.

Resumen

Muchos procesos importantes toman lugar en la nucleosíntesis de elementos. Uno de estos procesos es el decaimiento alfa. En este trabajo, el proceso decaimiento alfa se investiga en un ambiente astrofísico donde las condiciones extremas de temperatura presentes no pueden reproducirse en la Tierra. Para lograr esto, estudiamos los efectos de la temperatura en las vidas medias del decaimiento alfa. Utilizando un modelo simple basado en el tratamiento semiclásico del decaimiento alfa, las vidas medias son evaluadas considerando una partícula alfa que atraviesa la barrera de Coulomb generada por su interacción con la partícula hija. Encontramos las vidas medias dependientes de la temperatura para varios isótopos y las comparamos con lo resultados obtenidos en un trabajo anterior. Se concluye que incluir temperatura lleva a una reducción en las vidas medias.

Key words: Alpha decay, temperature, Q-value, half-life.

Palabras clave: Decaimiento alfa, temperatura, valor Q, vida media.

I. INTRODUCTION

It is well known that high temperatures modify reaction rates in many stellar processes (Hou et al., 2017). Elements heavier than Fe, such as Au or U are not produced in ordinary stellar nucleosynthesis, i.e., fusion reactions. They are produced by means of different processes occurring inside stars or during explosive events. The s-process, r-process and p-process are the main ways by which heavy elements can be synthesized (Iliadis, 2007). However, in the environments where heavy elements are synthesized, alpha decay is also taking place: A heavy isotope can be created but it is very likely that it decays as fast as it has been produced. Then, it is natural to think that the alpha decay process is modified by temperature too.

The importance of alpha decay theory resided in how well it can reproduce and/or predict the alpha decay widths and hence, the half-lives with the least number of parameters. Nowadays, most of the models require only the angular momentum number l of the parent state and the Q-value of the decay. However, effects such as electron screening, strong magnetic fields, non locality (Perez, Kelkar and Upadhyay, 2019) or temperature are not always taken into account in an alpha decay theory.

II. FORMALISM OF ALPHA DECAY

One of the most successful achievements of the quantum theory is the explanation of the alpha decay as a tunneling problem. This approach was developed independently by Gamow (Gammow,1928) and by Gurney and Condon (Condon and Gurney, 1928) in the late twenties. Since

[1]Corresponding author: je.perez43@uniandes.edu.co

then, many theoretical approaches have been proposed to describe the alpha decay, such as the shell model, the proximity model and the density-dependent cluster model.

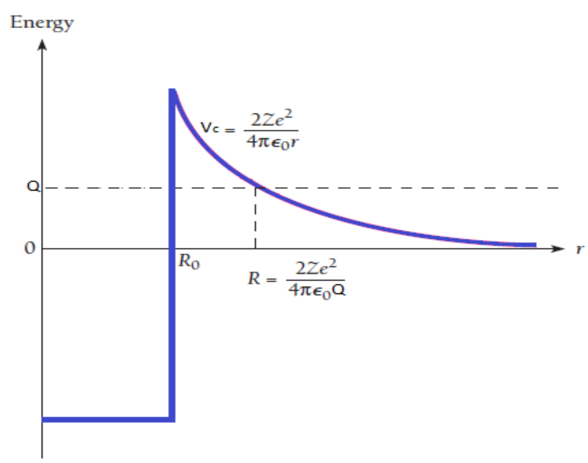
It has been found that the calculations of half-lives are in good agreement with experimental data, however, the theoretical values are very sensitive to the model chosen. Except for a multiplicative factor, corresponding to the preformation of the alpha particle inside the parent nucleus, the prediction of the half-lives, using the Gamow factor, is in good agreement with experiments. This disagreement is commonly attributed to the preformation probability which then differs in different model calculations. Since we want to make a comparison between the half-lives with and without the inclusion of temperature and not perform exact calculations of half-lives, a simple analytical formula is used to find the temperature dependent half-lives.

In our model (Beisser, 2003), we assume the existence of a preformed alpha particle inside the parent and consider the alpha decay as a tunneling of the alpha through the Coulomb barrier created by its interaction with the daughter nucleus. The total interaction potential is dependent on the relative distance between the daughter and alpha particle r and is given by

$$V(r) = V_n(r) + V_C(r) + \frac{\hbar^2 l(1+l)}{2\mu r^2} \tag{1}$$

where $V_n(r)$ and $V_C(r)$ are the nuclear and Coulomb potentials, respectively. The last term in equation (1) is the Langer modified centrifugal potential and μ is the reduced mass of the α-daughter system. Now, let us assume an incident alpha particle with energy Q which strikes on a rectangular potential barrier whose height $V_C(r)$ is greater than Q like in figure 1. In this scenario the nuclear potential vanishes. We also neglect the centrifugal contribution since we will restrict here to the simpler situation of alpha decay of spherical nuclei in the s-wave.

Figure 1.
Interaction potential between the alpha and the daughter nucleus.



Note: The strong potential is taken to be a rectangular well and the Coulomb potential is the one of two point particles.

The penetration probability, within a semiclassical approximation can be calculated as:

$$P = \exp\left[-2\int_{R_0}^{R} \kappa(r)dr\right] \tag{2}$$

where R_0 is the radius of the parent nucleus and R is the distance at which $V_C(r) = Q$. The wave number κ is given by

$$\kappa(r) = \left[\frac{2m_\alpha}{\hbar^2}(V_C(r) - Q)\right]^{1/2} \tag{3}$$

where m_α is the alpha particle mass. The radius R_0 and $V_C(r)$ are given by

$$R_0 = r_0 A^{1/3} \tag{4}$$

$$V_C(r) = \frac{2Ze^2}{4\pi\epsilon_0 r} \tag{5}$$

where $r_0 = 1.4fm$ and Z is the atomic number of the daughter nucleus. By using equation (2), equation (3), the electrostatic interaction (4) and after some simplifications one obtains:

$$P = \exp\left(2.97Z^{1/2}R_0^{1/2} - 3.95ZQ^{-1/2}\right) \tag{6}$$

Within a semiclassical approximation, the alpha decay width Γ is given by the product between the assault frequency ν with the potential walls and the penetrability probability P:

$$\Gamma = \nu P \tag{7}$$

The assault frequency is the inverse of the time required to traverse the distance back and forth inside the parent nucleus and can be expressed as $\nu = v/2R_0$, where v is the velocity of the alpha particle which in turn is related to the Q value as $v = (2Q/m_\alpha)^{1/2}$. Then, the alpha decay width is

$$\Gamma = \nu P = \frac{v}{2R_0}\exp\left(2.97Z^{1/2}R_0^{1/2} - 3.95ZQ^{-1/2}\right) \tag{8}$$

The half-life is related to the decay width as

$$t_{1/2} = \frac{\hbar \ln 2}{\Gamma} \tag{9}$$

III. TEMPERATURE DEPENDENT APPROACHES

The main objective of this work is to study the effect of introducing temperature in an alpha decay theory and compare it with a previous work. One can consider this to be a pilot study with a simple model in order to formulate a more detailed calculation in future. To achieve this, we introduce temperature in the Q-values by adding an *average excitation energy* $\bar{\epsilon}$ which is dependent on temperature. The effective Q-values are calculated using the equation

$$Q_{eff} = Q + \bar{\epsilon} \tag{10}$$

It is well known that the energy levels in a nucleus are not continuous but discrete, however, only a few first excited states are known in most of the cases. To find the average excitation energy we can use the available experimental information about the excited states together with an approximation for the continuum of states given by the nuclear level density $D(E)$.

The average excitation energy can be found using the usual statistical treatment, i.e.,

$$\bar{\epsilon}(A,Z,T) = -\frac{\partial}{\partial\beta}\ln Z(A,Z,T) \tag{11}$$

where A is the mass number of the parent nucleus and $Z(A,Z,T)$ is the canonical partition function as also given in (Davidson et al., 1994).

$$Z(A,Z,T) = \sum_{i}^{n} g_i \exp(-\beta E_i) + \int_{E_n}^{E_{max}} D(E)\exp(-\beta E)dE \tag{12}$$

where $g_i = 2J_i + 1$ and J_i is the spin of each i-state. E_n is the energy of the highest tabulated discrete level and $D(E)$ is the nuclear level density. We use the Ericson nuclear level density given by

$$D(E) = \frac{\sqrt{\pi}}{12}\frac{e^{2(aE)^{1/2}}}{a^{1/4}E^{5/4}} \tag{13}$$

The level density parameter a is given by $A/9$. If we take both discrete and continuum states, the average excitation energy is expressed as

$$\bar{\epsilon}(A,Z,T) = \frac{\sum_i^n g_i\, E_i\exp(-\beta E_i) + \int_{E_n}^{E_{max}} E \times D(E)\exp(-\beta E)dE}{\sum_i^n g_i \exp(-\beta E_i) + \int_{E_n}^{E_{max}} D(E)\exp(-\beta E)dE} \tag{14}$$

If we now change $Q \rightarrow Q_{eff}$ in equation (7), the decay width is expressed as

$$\Gamma = \frac{1}{2R_0}\sqrt{\frac{2Q_{eff}}{m_\alpha}}\exp\left(2.97Z^{1/2}R_0^{1/2} - 3.95ZQ_{eff}^{-1/2}\right) \tag{15}$$

As we will show later, the alpha-decay half-lives calculated using equation (14) considerably differ from the values found using the statistical approach, which is briefly explained below.

At high stellar temperatures ($T > 10^9 K$), Perrone and Clayton (Perrone and Clayton, 1971) proposed that alpha decay could be strongly dependent on temperature. This dependence would enhance the alpha decay process and hence, it would make the decays faster. In their work, the authors assumed that the decays are the result of thermally excited nuclear levels. What they found was that nuclei whose measured half-lives are billions of years then may decay in days or less when temperature is of the order of $10^9 K$. The temperature dependent half-life given by Perrone and Clayton (statistical model) is given by

$$[\mathrm{t1/2}(Z,A,T)]^{-1} = \int_0^\infty \sum_J \frac{F(Z,A,E,J,T)D(Z,A,E,J)dE}{t_{1/2}(Z,A,E,J)} \tag{16}$$

where $t_{1/2}(Z,A,E,J)$ is the temperature independent half-life for the decay of the parent nucleus to the daughter nucleus ground state. The last expression expresses the total decay of a nucleus as an integral over all energies above the ground state weighted by the nuclear level density $D(Z,A,E,J)$ and the occupation probability $F(Z,A,E,J,T)$, which can be written as

$$F(Z,A,E,J,T) = (2J+1)e^{-E/K_B T} \tag{17}$$

and represents the fraction of the ensemble populating a given state at a temperature T (K_B is the Boltzmann constant).

IV. RESULTS AND DISCUSSION

We use equation (14) together with equation (8) to calculate the temperature dependent half-lives for several temperatures. The results for different isotopes are given in table Table 1.

Table 1.
Temperature-dependent half lives

$t_{1/2}(T)[\mathrm{s}]$							
	Q	$0GK$	$0.8GK$	$1.2GK$	$1.6GK$	$2GK$	$2.4GK$
^{212}Po	8.954	3.7×10^{-6}	5.5×10^{-8}	4.9×10^{-8}	3.9×10^{-8}	2.7×10^{-8}	1.8×10^{-8}
^{174}Hf	2.494	1.7×10^{27}	4.3×10^{25}	1.6×10^{25}	4.6×10^{24}	9.0×10^{23}	9.5×10^{22}
^{144}Nd	1.903	4.3×10^{26}	1.3×10^{16}	1.2×10^{16}	8.2×10^{15}	3.2×10^{15}	6.5×10^{14}

Note: Calculated alpha-decay half-lives for several temperatures using the simple analytical formula [equation (14)] and effective Q-values given by equation (9). Half-lives are given in seconds.

We can see that as temperature increases, the half-lives decrease in all cases. In this calculations, we use the Ericson nuclear level density. The nuclear level density choice (and hence, the average excitation energy result) can slightly alter the way the half-lives change. Since temperature is addressed in the effective Q-values, it is important to note that even a small change in the Q-values will change the half-life, sometimes, by several orders of magnitude.

If we compare the reduction in Table 1. with that given in the paper of Perrone and Clayton, we notice that the reduction in their work is much more drastic than within the above model.

Both models show a general reduction of half-lives as the temperature increases. However, the statistical model shows a faster reduction in half-lives in comparison with the reduction obtained by using the analytical expression (14). For example, whereas the half-life of ^{174}Hf reduces by 5 orders of magnitude at a temperature of 2.4 GK here, one finds a reduction of 11 orders of magnitude in the work of Perrone and Clayton. This could be because of the foundations of the statistical model: Perrone and Clayton's statistical approach assumes that alpha emission can occur from all nuclear excited states. Equation (15) sums over all energies to calculate the temperature dependent half-lives. Due to the Maxwell-Boltzmann factor appearing in equation (16), the probability for an isotope to decay decreases as the energy increases. Although the contribution of the integral to the final half-life becomes smaller for high energies, the model assumes those energies are a continuum and that they would lead to an alpha decay. However, the energy levels of a nucleus are discrete. Moreover, it is found experimentally that the alpha decay process is very rare in isotopes which are in an excited level. The expression used by Perrone and Clayton for the penetration probability is similar to that used in the present work.

This investigation shows that alpha decay half-lives are reduced when temperature is included. The degree of this reduction strongly depends on the approach used. We point out the relevance of developing a model which takes into account this effect since alpha decay half-lives are an important ingredient in the calculation of the abundance of heavy elements. A better calculation within a more sophisticated model is planned for the future.

V. REFERENCES

Beisser, A. (2003), *Concepts of modern physics*. New York, U.S.A: Mc Graw Hill.

Condon, E.U., and Gurney, R.W. (1928). Wave mechanics and radioactive disintegration. *Nature, 51*, 204.

Davidson, N.J. et al. (1994). A semi-empirical determination of the properties of nuclear matter. *Nuclear Physics, A570*, 61c.

Gamow, G (1928). Quantum theory of the atomic nucleus. *Z.Phys,51*,204.

Hou, S.Q. *et al*. (2017). Non-extensive statistics to the cosmological lithium problem. *Astrophys. J,834*, 165.

Iliadis, C. (2007), *Nuclear physics of stars*. Chapel Hill, U.S.A: Wyley-VCH.

Perez, J.E., Kelkar, N.G. and Upadhyay, N. J. (2019). Assessment of nonlocal nuclear potentials in α decay. *Physical Review C, 99*, 024308.

Perrone, F.A. and Clayton, D.D. (1971). Thermally enhanced α-decay and the s-process. *Astrophysics and Space Science, 11*, 451.

ACKNOWLEDGEMENT

One of the authors (J. P. V.) thanks the Faculty of Science, Universidad de los Andes for financial support through the grant number INV-2019-67-1720. O.L.C acknowledges support from NSERC.