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TEMPERATURE-DEPENDENT CLUSTER DECAY HALF-LIVES

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Abstract

Temperature-dependent cluster decay half-lives are investigated based on the double folding model (DFM). The temperature dependence of the interaction potential is introduced through the charge and matter density distributions of the interacting nuclei, and the half-lives are calculated within a preformed cluster model. Subsequently, the temperature-dependent universal decay law (UDL) is fitted using the half-lives calculated with the double folding model.

Resumen

Los tiempos de vida media en la desintegración cluster, dependientes de la temperatura, son investigados con base en el modelo de doble plegado (DFM). La dependencia de la temperatura del potencial de interacción se introduce a través de las distribuciones de densidad de carga y materia de los núcleos interactuantes, y los tiempos de vida media se calculan dentro de un modelo de preformación de clusters. Posteriormente, se ajusta la ley de desintegración universal dependiente de la temperatura (UDL) utilizando los resultados obtenidos con el modelo de doble plegado

Keywords: Cluster decay, Double folding model, universal decay law, temperature-dependent half-lives.

Palabras clave: Desintegración cluster, modelo double folding, ley universal de desintegración, tiempos de vida media dependientes de la temperatura.

I. INTRODUCTION

A result of a type II supernova explosion is the creation and ejection of uranium and other heavy elements into space. Experimental observations provide a way to analyze the relative abundances of star material, but creating accurate models that can predict observations of heavy element abundance is a challenging task (Clayton, 1983; Iliadis, 2015). The main sources of the changes in element abundance are nuclear processes, mainly reactions, photodisintegrations, beta-decay, and alpha-decay. However, there exists the possibility of a small contribution from cluster decay, which in such astrophysical environments remains unexplored.

Sandulescu, Poenaru, and Greiner (1980) predicted a new kind of radioactivity in which the emitted particle is heavier than the alpha particle but lighter than the lightest fission fragment. This phenomenon is known as cluster radioactivity or cluster decay. The cluster decay was first observed with the emission of ^{14}C from ^{223}Ra (Rose & Jones, 1984). From then on, many other emitted clusters were detected namely, ^{20}O, ^{23}F, $^{24-26}Ne$, $^{28,30}Mg$, and ^{34}Si.

Many theoretical approaches employed to investigate α-decay have been extended to study the cluster emission. These models are based on the assumption of a preformed cluster of the decay products inside a parent nucleus, such that the decay happens due to the penetration of the light nucleus through the Coulomb barrier formed by the interaction of the light nucleus with the heavy daughter nucleus. The decay rate Γ or the half-life

$$t_{1/2} = \frac{\hbar \ln 2}{\Gamma} \tag{1}$$

is often calculated within the semiclassical JWKB approximation.

On the other hand, the empirical formula for the half-lives of radioactive decay processes, depending on some properties such as the Q value of the emitted particle, was first formulated in connection with α-decay by Geiger and Nuttall (1911). Different expressions have been tried to generalize the Geiger-

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Nuttall law to cluster decay (Royer, 2000; Poenaru et al. 2006). The linear formula of the logarithm of the cluster decay half-lives is known as the universal decay law (UDL). One such law was proposed by Qi et al. (2009):

$$\log \tilde{t}_{1/2} = a\chi' + b\rho' + c \qquad (2)$$

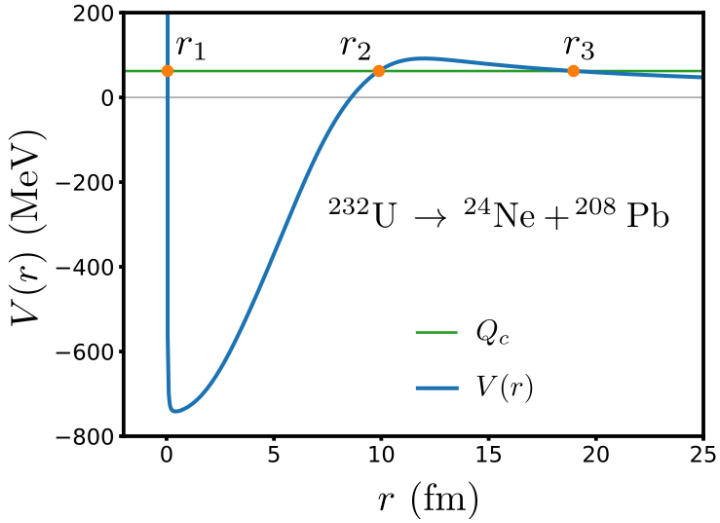
where $\tilde{t}_{1/2}$ is the dimensionless cluster decay half-life, $\chi' = \mathrm{Z_c Z_d}\sqrt{\mathrm{A/Q_c}}$, $\rho' = \sqrt{\mathrm{AZ_c Z_d}\left(\mathrm{A_d^{1/3}} + \mathrm{A_c^{1/3}}\right)}$ and the values of the parameters a, b, and c are determined using a fitting procedure. $\mathrm{Z_c}$, $\mathrm{A_c}$, $\mathrm{Z_d}$, and $\mathrm{A_d}$ are the atomic and mass numbers of the light nucleus (referred to as cluster here) and the daughter nucleus, respectively.

Before delving into a detailed investigation of cluster decay under the high-temperature and high-density conditions characteristic of astrophysical environments, it is beneficial to explore the sensitivity of the universal decay law (UDL), specifically the coefficients in this law, to temperature. To achieve this objective, the present work focuses on the temperature dependence of cluster decay and is organized as follows. Firstly, we introduce the double-folding model (DFM) utilized to evaluate the decay half-lives within a semiclassical approach to the tunneling problem. The temperature dependence in the DFM is introduced through temperature-dependent matter and charge density distributions. Next, the parameters in the UDL are fitted to reproduce the temperature-dependent DFM half-lives. Finally, we discuss the temperature dependence of the parameters thus obtained.

II. FORMALISM

The decay width in Eq. (1) is given within the semiclassical JWKB approximation (Gurvitz & Kalbermann, 1987; Kelkar & Castañeda, 2007) as

Figure 1
Total potential for cluster-daughter nucleus tunneling problem.



Note: The cluster-daughter interaction potential V(r) for the decay ^{232}U → ^{24}Ne+^{208}Pb. The three classical turning points determined by $V(r_i)$=Q_c are pointed out.

$$\Gamma = S_c \frac{\hbar^2}{2\mu}\left[\int_{r_1}^{r_2} \frac{dr}{k(r)}\right]^{-1} \exp\left[-2\int_{r_2}^{r_3} k(r)d\,r\right] \quad (3)$$

where S_c is the preformation factor, μ is the reduced mass of the cluster and the daughter nucleus system, the exponential factor is the penetration probability and the factor in front of it is the normalization of the bound-state wave function in the region between r_1 and r_2, the first and second turning points that are solutions of $V(r_i)$=Q_c (as shown in Fig. 1). The wave number is given by $k(r) = \sqrt{\frac{2\mu}{\hbar^2}[\,|V(r) - \mathrm{E}|\,]}$ where E is the energy of the tunneling particle (taken to be the Q_c value here). The interaction potential $V(r)$ is composed of the nuclear potential $V_N(r)$, the Coulomb potential $V_C(r)$, and the centrifugal barrier with the Langer modification to ensure the correct behavior of the JWKB wave function near the origin (Langer, 1937). The total potential is then given by

$$V(r) = \lambda\, V_N(r) + V_C(r) + \frac{\hbar^2\left(l+\frac{1}{2}\right)^2}{2\mu r^2}, \quad (4)$$

where r is the separation between the center of masses of the cluster and the daughter nucleus, and λ is the strength of the nuclear interaction and is fixed by imposing the Bohr-Sommerfeld quantization condition,

$$\int_{r_1}^{r_2} k(r)d\,r = \left(n+\frac{1}{2}\right)\pi. \quad (5)$$

where n is the number of nodes of the quasibound wavefunction of cluster relative motion.

The nuclear potential is calculated using a realistic nucleon-nucleon (NN) interaction folded with the density distributions of both interacting nuclei (Kelkar & Castañeda, 2007; Kelkar & Nowakowski, 2016; Perez Velasquez et al. 2019). The nuclear and Coulomb potentials are obtained within the double-folding model (Satchler & Love, 1979) as

$$V_N(\boldsymbol{r}) = \int d\boldsymbol{r_1}\, d\boldsymbol{r_2}\, \rho_c(\boldsymbol{r_1}) v(|\boldsymbol{s}| = |\boldsymbol{r_2} + \boldsymbol{r} - \boldsymbol{r_1}|)\rho_d(\boldsymbol{r_2}). \quad (6)$$

where $v(|\boldsymbol{s}|)$ is M3Y-Reid-type nucleon-nucleon interaction

$$v_N(\boldsymbol{s}) = 7999\frac{\exp(-4|\boldsymbol{s}|)}{4|\boldsymbol{s}|} - 2134\frac{\exp(-2.5|\boldsymbol{s}|)}{2.5|\boldsymbol{s}|} \quad (7)$$

where $|\mathrm{s}| = |\mathrm{r} + \mathrm{r}_2 - \mathrm{r}_1|$ is the distance between a nucleon in the daughter nucleus and a nucleon in the cluster. The Coulomb potential is calculated in a similar way as Eq. (6) using the charge density distributions $\rho^{\mathrm{C}}(\boldsymbol{r})$ and the standard proton-proton interaction.

In order to introduce temperature dependence, the matter and charge density distributions of the nuclei can be calculated as (Antonov et al., 1989; Gupta et al., 2007; Aygun, 2019),

$$\rho(r,T) = \rho_0(T)\left[1 + \exp\left(\frac{r - R_i(T)}{a_i(T)}\right)\right]^{-1}, \quad (8)$$

where $\rho_0(T)$ is obtained by normalizing and the thermal effects are introduced as (Gupta et al., 2007)

$$R_i(T) = R_{0i}(T=0)[1 + 0.0005T^2], \tag{9}$$

and

$$a_i(T) = a_i(T=0)[1 + 0.01T^2]. \tag{10}$$

where $\mathrm{R_{0i}} = 1.07\mathrm{A_i^{1/3}}$ fm and $a_i = 0.54$ fm.

The temperature dependent cluster decay half-lives are calculated for different isotopes, namely,

$$^{228}\mathrm{Th} \rightarrow {}^{20}\mathrm{O} + {}^{208}\mathrm{Pb},$$
$$^{232}\mathrm{U} \rightarrow {}^{24}\mathrm{Ne} + {}^{208}\mathrm{Pb},$$
$$^{236}\mathrm{Pu} \rightarrow {}^{28}\mathrm{Mg} + {}^{208}\mathrm{Pb},$$
$$^{242}\mathrm{Cm} \rightarrow {}^{34}\mathrm{Si} + {}^{208}\mathrm{Pb}.$$

The results on the calculated half-lives of those isotopes at different temperatures allow us to refine the universal decay law (UDL) in Eq. (2) including the temperature dependence within the parameters a, b, and c. The UDL is then rewritten as

$$\log \tilde{t}_{1/2}\,(T) = a(T)\chi' + b(T)\rho' + c(T), \tag{11}$$

where

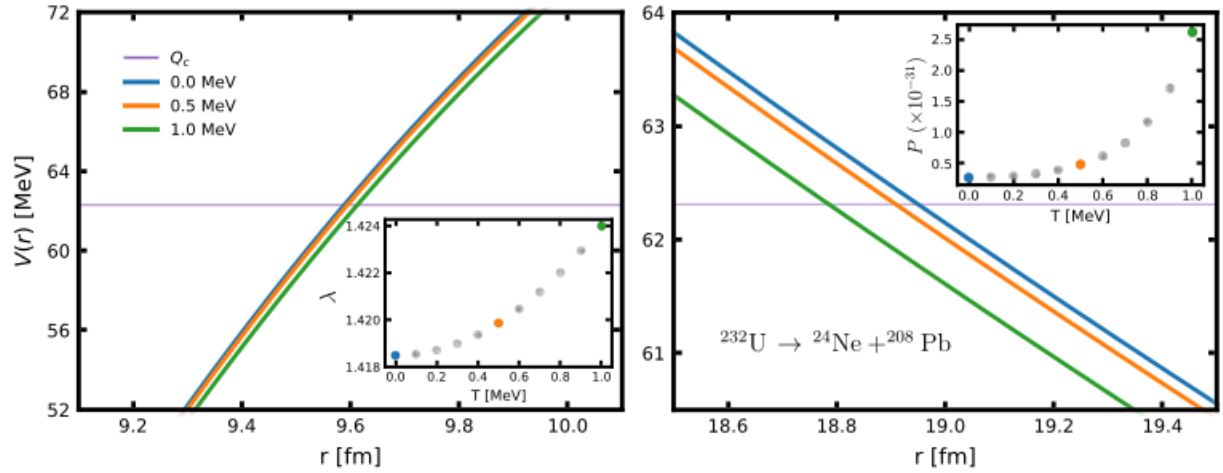
$$\begin{aligned} a(T) &= a_0 + a_1 T + a_2 T^2, \\ b(T) &= b_0 + b_1 T + b_2 T^2, \\ c(T) &= c_0 + c_1 T + c_2 T^2. \end{aligned} \tag{12}$$

The temperature dependent parameters $a(T)$, $b(T)$, and $c(T)$ are fitted from the outcome of the cluster decay half-lives using the temperature dependent double folding model. An improved version of this simple UDL formula with a deeper analysis of the temperature dependence has been presented in (Rojas-Gamboa et al 2022).

III. RESULTS AND DISCUSSION

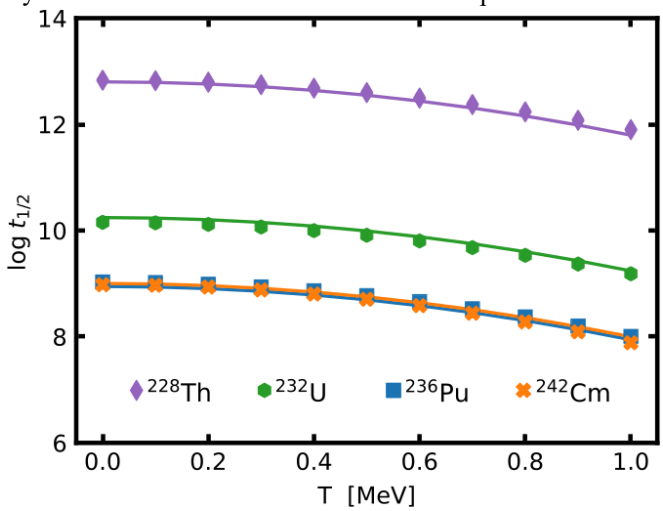
To investigate the effect of temperature in cluster decay, we numerically calculate the half-lives of some even-even nuclei using the double folding model (with Γ evaluated as in Eq. (3) but assuming $S_c = 1$). The temperature dependence is incorporated into the charge and matter density distribution through the half-density radius, as given in Eq. (9), and the surface diffuseness, as given in Eq. (10). Thi temperature dependence causes a shift in the positions of the second and third turning points, which are solutions of $V(r_i) = Q_c$. Here, the total potential $V(r)$ is obtained using Eqs. (4) and (6), with the strength of the nuclear potential λ fixed at each temperature by the Bohr-Sommerfeld quantization condition, as expressed in Eq. (5).

Figure 2
Influence of Temperature on second and third turning points.



Note: The interaction potential in the region of the second (left panel) and third (right panel) turning points for the decay $^{232}\mathrm{U} \rightarrow {}^{24}\mathrm{Ne}+{}^{208}\mathrm{Pb}$ at different temperatures. Left inset: λ values as a function of T. Right inset: Penetration probability as a function of T.

Figure 3
Logarithm of the cluster decay half-lives in seconds as a function of temperature.



Note: Logarithm of the cluster decay half-lives in seconds as a function of temperature. The symbols correspond to double folding calculations and the lines correspond to the universal decay law fitted from it.

In Fig. 2, the shifting of those turning points for the cluster decay $^{232}\mathrm{U} \rightarrow {}^{24}\mathrm{Ne}+{}^{208}\mathrm{Pb}$ at different temperatures, namely 0, 0.5, and 1.0 MeV, is presented. The insets of Fig. 2 show the values of λ and the penetration probability Pas a function of temperature. Even though the strength of the nuclear part is stronger when the temperature increases, the reduction in the width of the Coulomb barrier leads to an increase of the penetration probability. The shift in the turning points with temperature is not large, but the penetration probability being an exponential factor is sensitive to this change and grows rapidly with decreasing width of the barrier. Therefore, the cluster decay half-lives decrease with temperature. This effect is depicted in Fig. 3, which shows the reduction of the half-lives when the temperature increases. In Fig. 3, the temperature-dependent universal decay law (solid lines) of cluster decay fitted from the

temperature-dependent double folding potential (symbols) is also shown. The temperature dependence of the universal decay law, Eq. (2), is included in the parameters a, b, and c as in Eq. (12). This constitutes an easy way to obtain the half-life at each temperature. We find that $a(T)$ and $b(T)$ are constant and the temperature dependence is all contained within $c(T)$. The values of the non-zero coefficients are $a_0 = 0.60995$, $b_0 = -0.02942$, $c_0 = -19.24970$, $c_1 = -0.00365\, MeV^{-1}$, and $c_2 = -1.00106\, MeV^{-2}$. All information about the cluster decay is contained inside χ' and ρ', but it is necessary to investigate how the coefficients $a(T)$, $b(T)$, and $c(T)$ depend on temperature in order to obtain the temperature dependent half-lives. This study allows us to understand the impact of temperature on the cluster decay in a range of temperatures relevant for astrophysical phenomena, providing us with the motivation to further investigate the temperature dependent decay with a more sophisticated framework.

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