HP PRIME: product of research in Mathematics Education[1]

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Abstract

The development of the HP Prime graphing calculator and wireless classroom was aided significantly by a core group of mathematics educators who advised the development team and a pedagogical architect who sat on the team. This paper reflects back on the design decisions they made and the research basis that shaped at least some of those decisions.

Key words

Graphing calculator, mathematical fidelity, representational fluency, instrumental genesis.

Resumen[2]

El desarrollo de la calculadora gráfica HP Prime y el aula inalámbrica fue ayudado de manera significativa por un grupo de educadores matemáticos que asesoró al equipo de desarrollo y por un arquitecto pedagógico quien participó con el equipo. En este trabajo se refleja de nuevo sobre las decisiones de diseño que han hecho y de los fundamentos de la investigación que dio forma a por lo menos algunas de esas decisiones.

Palabras clave

Calculadora gráfica, fidelidad matemática, fluidez representacional, génesis instrumental.

1. Overview

> …major limitations of computer use in the coming decades are likely to be less a result of technological limitations than a result of limited human imagination and the constraints of old habits and social structures. (Kaput, 1992)

We start with an observation that may be outside the realm of mathematics education research but is of interest nonetheless, as it shows the development team's appreciation of the difficulties inherent in bringing mathematics education technology to life. We make a fundamental distinction between technology-based mathematical tools for

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learners and those for doers. We recognize that any individual is, at any given point in time, part learner and part doer; however, experience has shown us that doers want flexibility, efficiency, and power while learners need structure and help. We categorize doers as those who have a well-formed and well-connected web of concepts and procedural knowledge in their subject; conversely, we categorize learners as though whose webs of concepts and procedural knowledge show isolation, holes, and errors. This distinction is not meant as a definition for researchers but rather as a guiding principle for the development team.

It was clear that we were creating a solution for learners and not for doers. The high-level model of the graphing calculator needed to reflect the fact that it was intended for learners. We decided on the following characteristics for our model:

- The model reflects the nature of the mathematics the learner will study; that is, it has a high degree of mathematical fidelity
- It is modular and extensible
- It is an internal constraint in the sense of Trouche (2005); that is, it is fundamental to the machine and the user cannot change it

2. HP Apps and Views

In the end, our search for the high-level model took us back to a product HP developed in 1993. This model consists of a set of apps to compartmentalize related functionality, along with a Home screen for numerical computations and a corresponding CAS screen for exact or symbolic computations. The concept for the HP Prime app is based on the HP 38G applet concept, which was originally conceived of as a sandbox. A sandbox is a bounded area containing sand, a shovel, and a pail. Thus it is safe, simple and self-contained, with a few relevant tools ready to hand. You can take things out of the sandbox and bring other things in as you see fit, but the original sandbox just has sand, shovel, pail, and a boundary. The breakthrough for the HP 38G applets (and our HP Prime apps) came with the realization that the app-as-sandbox could have 3 views that map to the ubiquitous symbolic, graphic, and numeric representations of mathematical objects. That is, the realization was made that the graphing calculator dealt essentially with mathematical representations and that the predominant representations in school mathematics were these three. If the apps each dedicated one view to each of these three, in a uniform and systematic way, then the organization of the calculator could be vastly simplified. Thus the keyboard has an App key to select an app and three keys next to it (Symb, Plot, and Num) to present these three views. The result is shown in Figure 1 below.

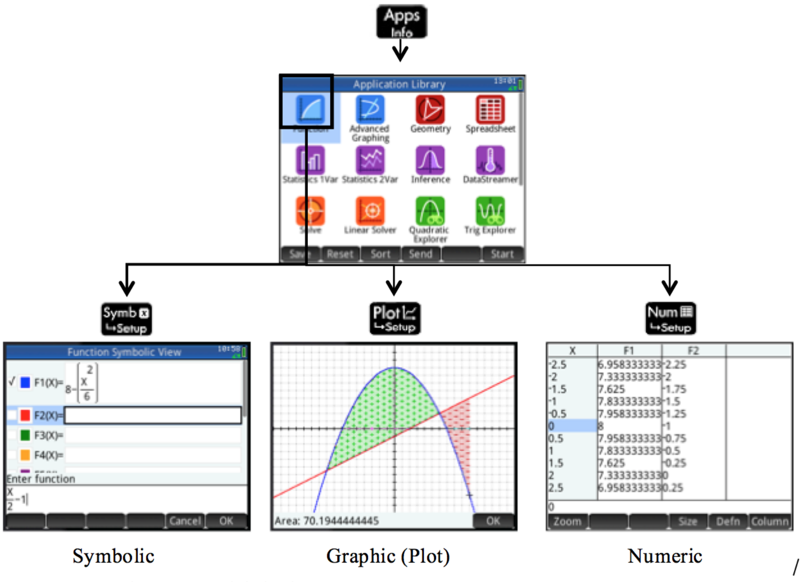

Figure 1: HP Prime apps and their views.

The vision was that students would spend most of their time in the 8 most common apps (Function, Geometry, Statistics, etc.); we were able to make the 3 views almost identical for 5 of these 8 apps and only slightly different for the other 3 apps. The result was a model that mapped very well to the NCTM Standards, provided a simple structure for the learner, and was accessible to anyone familiar with their own smartphone. Our conjecture was that this model could have a significant impact on instrumental genesis.

Next we look at a few of the HP Prime apps as examples.

3. The Advanced Graphing App

> Through the use, interpretation, and coordination of multiple (external) representations teachers and students communicate about mathematical entities; when a broad range of representations is coupled with representational fluency, the opportunity for building rich meaning grows. ... Because technology has the potential for broadening the representational horizon, consideration of representation is fundamental to the development of educational technologies that support problem-solving, learning, and learner-centered design issues. (Zbiek, Heid, Blume, & Dick, 2007)

For too long, the graphical representations afforded by graphing calculator technologies have focused on the representations of functions and even these have enjoyed limited mathematical fidelity. For example, what many students (and some teachers) perceive as the drawing of a vertical asymptote in the graph of $y = \tan(x)$ on many graphing calculators is actually an example of a lack of mathematical fidelity. With the Advanced Graphing app, we wanted to "broaden the representational horizon" and at the same time, increase both the mathematical and cognitive fidelity of these graphical representations. To do so, we collaborated with Jeff Tupper to create an app that would use an interval arithmetic approach to creating graphical representations from symbolic statements: the Advanced Graphing app.

This new approach brought a variety of benefits. First, one is no longer limited to graphing functions and the familiar polar and parametric types of equations. Rotated conics, inequalities, regions, and all sorts of other new graphical representations are now quite easily available. In all of these graphical representations of a complete symbolic mathematical statement, the fundamental and universal concept is that the graph presents all the ordered pairs within the viewing window (that the machine can represent) which make the mathematical statement true.

In addition to expanding the set of statements which can be graphed, the Advanced Graphing app frees the user from constraints related to the forms of mathematical statements accepted by the machine. For example, a linear graph can be represented as $y = m \cdot x + b$, $y - y_0 = m(x - x_0)$, $\frac{x}{A} + \frac{y}{B} = 1$, or $A \cdot x + B \cdot y + C = 0$, etc. This freedom allows the user to keep the symbolic representations as they have been presented, without having to change the form to accommodate constraints of the technological tool. This freedom from constraint, in turn, reduces the complexity of instrumental genesis.

Finally, the interval arithmetic approach improves the mathematical and cognitive fidelity of these graphical representations in the sense that the display pixel always remains a region. Many current graphing calculators subtly imply to the user that each pixel on the graph is a point. For the HP Prime Advanced Graphing app, however, this is not the case. Resolution of the graph does not proceed from left to right and pixel by pixel but rather from larger rectangular regions to smaller ones, until the resolved regions are no more than the size of the pixels. The promise that the graphical representation makes is that the region represented by a lit pixel contains at least one point that satisfies the mathematical statement being graphed. This point is quite subtle and may not be readily observed by the user, but it is reflected in a higher degree of mathematical fidelity in the graphical representation of pathological functions, as shown in Figures 2-3 below.

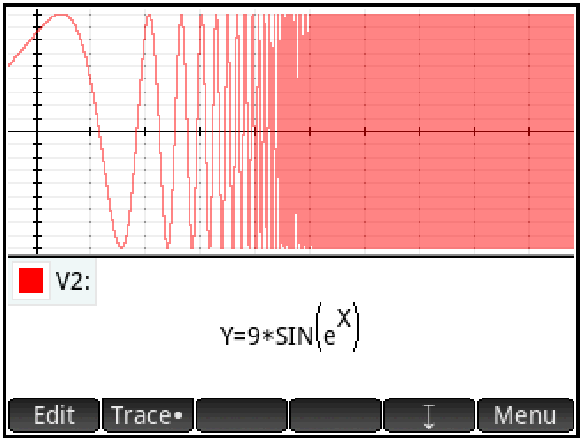

Figure 2: $y = 9 \sin e^x$.

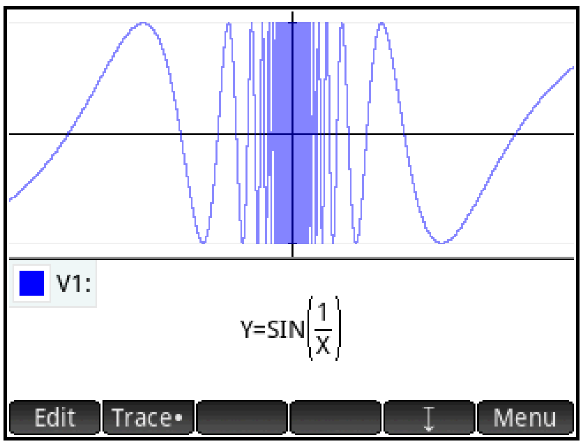

Figure 3: $y = \sin \frac{1}{x}$.

As mentioned earlier, the app can handle mathematical statements that do not represent functions at all. This brings up questions regarding the Numeric view for such mathematical statements, as well as how the tracer should function in Plot view. One innovation in this area is that the tracer in Plot view (as well as the table in Numeric view) can be restricted to points of interest only (extrema, intercepts, inflection points, intersections, etc.). In this way, the student can more easily focus on properties of a small subset of important numerical values rather than trying to first determine which values in a relatively large set of numbers are actually of interest.

4. The Geometry App

The Symbolic-Plot-Numeric structure of the HP Prime apps gives users a significant entry point for instrumental genesis in the Geometry app. For each geometric object created in Plot view, there is a defining symbolic entry created in Symbolic view. These Symbolic view entries can be viewed by the user. By viewing a Symbolic view entry, the user can learn the syntax and function of many of the dynamic geometry commands. In fact, the user can edit these symbolic definitions as well, and the Plot view will display those changes accurately. In this way, the user can discover exactly how a geometric sketch was created, explore related sketches with differing definitions, and create new objects without using the Plot view user interface at all.

5. Touch Gestures, Representational Fluency, and Fidelity

> For our purposes, representational fluency includes the ability to draw meaning about a mathematical entity from different representations of that mathematical entity, and the ability to generalize across different representations. Our operational definition aligns with the notion of representational versatility that Hong and Thomas (2002) took "to include both fluency of translation, and the ability to interact procedurally and conceptually with individual representations" (p. 1002). (Zbiek, Heid, Blume, & Dick, 2007).

We propose that representational fluency is more easily attained when students can directly interact with representations via a variety of touch gestures, such as pinch, drag, flick, etc. Dynamic geometry applications have received considerable approbation for allowing a variety of mediated user interactions with representations via a mouse. HP Prime removes the mediation of the mouse and allows direct touch interactions in the Geometry app, as well as extending those direct touch interactions to other graphical environments. For example, establishing local linearity of a function at a point by zooming in smoothly on that point using a dynamic pinch gesture is a fundamentally different technological experience than zooming in by discrete stages using a zoom menu option. With the pinch gesture, there is a visceral aspect to the experience that is lacking with the zoom menu method. For another example, exploring the limit of a function at a point by zooming in on the row of the table that represents that point using a dynamic vertical pinch gesture is again a fundamentally different experience than changing the step value in the table by discrete stages using a menu. In both of these examples, meaningful student interactions with mathematical representations are achieved with few command or organization constraints in the sense of Trouche (2005).

In the development of HP Prime, we took great care to imbue common touch gestures with mathematical meanings across multiple representations; the result is a high degree of both mathematical and cognitive fidelity.

6. Example: Scaling the Viewing Window

> A well-known example of an instrumented action scheme with related conceptual and technical aspects concerns scaling the viewing window of a graphing calculator (Goldberg 1988). An instrumented action scheme that needs to be developed involves the technical skills of setting the viewing window dimensions, but also the mental skill to imagine the calculator screen as a relatively small window that can be moved over an infinite plane, where the position and the dimensions of the window determine whether we hit the graph. We conjecture that it is the incompleteness of the conceptual part of such a scheme that causes the difficulties that many novice graphing calculator users have with setting appropriate viewing screens. (Drijvers & Gravemeijer, 2005).

In the Plot view of an HP Prime app, simply dragging the finger in the intended direction pans the viewing window accordingly. We conjecture that this touch gesture introduces the conceptual component of the instrumented action scheme in a way that leads naturally to the more technical skill of setting the window boundaries manually via a menu. Similarly, pinching to zoom in and out on the viewing window conveys an important aspect of the concept of the viewing window having variable area with respect to the infinite plane, even though the physical dimensions of the calculator screen never change. The separate technical skill of deciding on boundary values for the viewing window then becomes a useful additional skill, but not one that blocks the user from developing the essential conceptual understanding that drives the instrumented action scheme.

7. Instrumental Orchestrationsand the Wireless Classroom

> …we have introduced the notion of instrumental orchestration to refer to an organization of the artifactual environment, that an institution (here the schooling institution) designs and puts in place, with the main objective of assisting the instrumental genesis of individuals (here students). (Trouche 2005)

We were aware that the successful introduction of the HP Prime graphing calculator depended on more than its own design, regardless of that design's potential for reducing the complexity of instrumental genesis, providing representations with a high degree of mathematical and cognitive fidelity, and facilitating representational fluency. It still had to be adopted by teachers and shown to be viable in the classroom. These requirements introduced the need for models of use; that is, exemplars of proven classroom value. While we did not want to create such models ourselves, the total classroom solution had to lend itself well to the design and execution of such models. These models had to take into account such variables as the size and shape of the physical classroom, the set of artifacts within that classroom (including other technologies), and the mathematical situations and workflows typically present in that classroom (with their attendant socio-cultural norms). The goals here are thus not only to assist students in developing both instrumental and mathematical knowledge, but also to assist the teacher (and by extension the teacher of teachers) to develop and execute exemplary instrumental orchestrations. We made the decision to focus on instrumental orchestrations in the face-to-face classroom, privileging that scenario over any distance-learning scenario. We then took a step back and identified a small set of meta-workflows common to as many models as we could imagine. These meta-workflows are as follows:

1. Teacher distributes to students materials necessary for investigating a mathematical situation
2. Students create and interact with representations, extracting meaning from these interactions and reflecting on their implications in the context of the mathematical situation
3. Students (and teacher) engage in mathematical discourse, mediated by sharing one or more external representations
4. Students (and teacher) engage in instrumental discourse, mediated by sharing one or more artifacts
5. Teacher assesses robustness of one or more student instruments
6. Teacher assesses student understanding of the mathematical situation

Although none of these meta-workflows specifically require a wireless classroom network, we became convinced that a well-designed wireless network could add considerable value simply by distributing materials, providing a shared representation or artifact (shared focus of attention), and facilitating formative assessment of student understanding. The wireless network, in short, would focus on the social aspects of learning mathematics via technology, but with the realization that this network exists in a classroom consisting of students and their teacher who are face-to-face. Our wireless network was designed to have the following characteristics, most of which support the established meta-workflows:

- The network cannot require technical knowledge to establish; that is, it must be possible for a non-technical teacher to set it up
- The network must acknowledge and defer to the face-to-face classroom experience; that is, it does not force the teacher to use the wireless network for tasks better accomplished face-to-face (privileges the face-to-face scenario)
- It saves the teacher time in the distribution of technology-based materials (#1)
- It facilitates formative assessment of student progress via polling and messaging (#5, #6)
- It encourages mathematical and instrumental discourse by allowing students and teachers to share representations, thus providing a shared focus of attention (#3, #4)

These design decisions were implemented in the HP Prime Wireless Classroom Network. Set up requires three steps on the part of the teacher (install the software, plug in 1 or 2 antennae, and name each antenna's network) and two steps on the part of each student (plug in the wireless module and then follow a 4-tap process to select a network). After that, connection is automatic. In our pilot test sites, establishing the network was reported to be remarkably easy. So far in our test sites, the teachers report using the network mainly for distributing materials and for sharing a student's HP Prime display to the class for discussion purposes. Both of these uses were prominent in our meta-workflows.

8. Conclusion

The development of HP Prime and the HP Wireless Classroom Network was guided by a small group of teachers and a pedagogical architect, both of whom are familiar with current research in mathematics education. The result holds considerable promise for reducing the complexity inherent in instrumental genesis, increasing the mathematical and cognitive fidelity of technology-based representations, facilitating representational fluency, and developing successful instrumental orchestrations via a wireless classroom network.

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