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THE PROBLEM OF TRANSFORMING VALUES INTO PRICES

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RESUMEN

El objetivo de este documento es presentar y analizar en una forma integrada las soluciones alternativas propuestas para el problema de transformar los valores trabajo en precios. Considerando que la literatura en este tema es dispersa en cuanto a las presentaciones al problema y sus soluciones, en este documento se sigue una presentación matemática que permita integrar dichas presentaciones en una misma terminología. Especial énfasis es puesto en la presentación de un ´nuevo´ procedimiento de normalización que busca enfatizar el rol de la teoría del valor trabajo como una teoría de formación de valor más bien que formación del precio. En esta tradición, los precios tenderán a asignar o distribuir valor entre los agentes económicos. Contrario a esta postulación, el documento también valora la solución clásica que se enfoca en una determinación simultánea de los precios y el valor. En este sentido el documento busca un tratamiento integrado de las soluciones propuestas, para poder comprender también las divergencias.

ABSTRACT

The object of this paper is to describe and analyze in a formal and integrated way the alternative solutions proposed to the problem of transforming labor values into

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prices. Considering that the literature on this topic is sometimes diffused, this paper presents in a unified way the solutions advanced following a mathematical treatment and presentation. Special emphasis is paid to a ´new´ normalization procedure that emphasize on the role of the labor theory value as a theory of value formation rather than price. In this tradition, prices will tend to assign or distribute value across society. As opposed to this approach, the paper also contrasts and analyzes the classical approach that focuses on the simultaneous determination of price and value. The paper is then a search for a unified treatment of the topic.

I. INTRODUCTION

The labor theory of value (LTV) plays a central and controversial role in Marxian Economic Analysis. The assertion that value is created in production and transferred in exchange is fundamental and has been a central concept for many different authors that have developed some Marxian categories.

At the core of the transformation problem underlies a tension toward the role of the LTV. As Foley (1982, page 41) affirms [sic] "in a certain sense, the choice of which of the propositions listed above should be maintained in generalizing the labor theory of value to situations where prices of commodities are not proportional to labor values expresses an understanding of what the labor theory of value means at its core. This is why seemingly technical controversies over the "transformation problem" had so much life and energy; they are in fact controversies over the meaning and coherence of the labor theory of value."

If the organic composition of capital is the same for all sectors in the capitalist economy then the quantity of abstract labor time embodied and commanded by one specified commodity is of the same proportion. In the one hand, the value of all commodities produced in society reflects the quantity of abstract labor spent in the production of these commodities. In the other hand, the total price for this production of new values is exactly the same value produced by total labor. Moreover, since the organic composition of capital is the same for all sectors then one has that individual prices are proportional to individual values and, looking at value as the quantity of labor directly and indirectly spent in the production of the commodity, then one obtains that there is no "sistematic" value deviation from prices. One has either Marx's, Ricardo's or even Solow's argument of working in a 'big factory' where values are produced under the same proportion between variable capital and constant capital for all sub-factories.

Marx derived from his conceptualization of the LTV that at the aggregate level total price should equal total value and total surplus should equal total profits. One could derive from this statement that at the individual level, the quantity of abstract labor embodied in the commodity might not reflect the same quantity of abstract labor commanded. Still, the point is that at the aggregate level this un-balanced condition should not hold. There should be conservation at the aggregate level. It is the breaking of this conservation law, or in other terms, the falling apart of the above two conditions i.e., 1) total price equals total value, 2) total profit equals total surplus which originates what nowadays is known as the Transformation Problem (TP). The possibility of breaking these conditions apart is obtained once different organic compositions of capital are allowed and one defines value as the direct and indirect quantity of abstract labor time spent in the production of the commodity, the approach followed by the classical tradition.

The object of this paper is to analyze the controversy paying special attention to a ´new´ approach called as a new normalization procedure. The object is to emphasize how this controversy has risen a discussion toward the interpretation of the LTV where the notion of value to some extent has been separated from the concept of price. Prices will tend to assign or distribute value across society. This is not the case of the classical approach which hinges on the mixture of price and value simultaneously determined. For this new approach, the LTV is directed toward a conservation law not necessarily related to an accidental theory of price. Prices like money are a form of value. It is by following this fundamental principle that a serious attack is advanced toward a second fundamental issue from classical Marxism: variable capital. Variable capital according to these authors has being reduced to a technical coefficient, and the notion of class conflict driven out of the center of analysis by means of an augmented matrix and its maximum eigenvalue.

I structure the paper in three sections. The first section develops in general terms Foley-Dumenil's conception of value and exploitation and brings by these means their reading and interpretation of the LTV. The general character of price theories associated to this reading of LTV is analyzed. The second section opens with the analysis of Lipietz of price and value under a Dumenil-Foley (D-F) conception of value. This view allows the study of the relation between this new approach and the specific theory of price. Using the uniform rate of profits approach, I study the determination of the system and the implications for the Marxian conditions proposed. Finally, I present Morishima analysis as a contrast to our previous case and as a mean to explore deeper both conceptions of the LTV once Lipietz model has been set up. Solutions to the model and implications are studied to obtain a better framework of contrast to the Lipietz's model and D-F's in general. The iterative process is studied to see its contrast and similitude to the initial position of Morishima and the older tradition started from Bortkiewicz. I conclude with a series of comments toward the notion of the LTV and the value of labor power in both traditions and further developments for the exogenous variables determination of both models.

II. THE MODEL OF DUMENIL-FOLEY

In the one hand there is a tradition that pays special emphasis on a LTV focusing on the creation of value rather than its circulation. For this tradition lead by Dumenil (1980) and Foley (1982), Marx's main assertion on the labor theory of value is that the net product created by society is the result of the total labor spent during the same period. Thus the net product should reflect the total quantity of labor spent during that period of time. "The value added of the mass of commodities produced is the total quantity of labor expended in producing them" (Foley, (1982), 14). Total price equals total value by definition.

Furthermore, there is a need of an explicit treatment of the surplus value. A fundamental object of this approach is to employ the LTV to explain the notion of exploitation in a capitalist system, i.e., how surplus value is created during the process of production. All value is created in production but not transferred as wages. In order to obtain this result and allow simultaneously for the condition of total surplus value equal total profits they consider the rate of surplus as given either from total profits to total wages or the value of money as a proportion of total labor to total value added. Henceforth, the value of labor power is obtained as a proportion of total surplus per unit of labor to the surplus rate and then total profit will equal total profits.

Consider this problem in mathematical terms as follows. There are two alternative ways to approach this problem. One way follows Foley and defines the value of money as the proportion between total labor employed and total value added. A second alternative follows Lipietz (1982) and defines an appropriate value of money and a given surplus rate to work out the implications of the theory. Both directions tend to lead one to the same result but, as will be argued below, Lipietz's presentation is rather less general than Dumenil-Foley's reading of Capital.

Let one define X and Y as gross and net output respectively, L as total quantity of labor employed during the period and P as the price vector for each commodity. It must be emphasized that there is no ad hoc specification for any price theory. Following, one has

$$\lambda_m = \frac{LX}{PY} = \frac{\Lambda Y}{PY}$$

where γ_m is defined as the value of money, that is, how many units of labor buys a unit of money.

Having λ_m one can define the value of labor power w^* from the money wage rate , as follows,

$$w \cdot \lambda_m = w^*$$

which expresses how many units of labor can the money wage rate buy.

Lastly, one can show that both TP conditions are obtained,

$$\lambda_m = \frac{LX}{PY} = \frac{\Lambda Y}{PY}$$

$$\begin{cases} \text{total profits} = PY - wLX \\ \text{total surplus} = (PY - wLX)\lambda_m \end{cases} \Rightarrow \lambda_m = \frac{\text{total profits}}{\text{total surplus}}$$

Thus, by fixing the value of money as the proportion of total labor employed to total value added in price terms, both conditions directly follow. Several issues must be emphasized.

First, the main assertion that total labor always represents total value added.

Second, there is no specification for any price theory at all. Prices can be obtained previously to our analysis. Whether or not these prices happen to be proportional to values is not an important issue here; this condition only means that there is a redistribution of value as Marx (1981) argued.

Third, one can define the value of labor power by using either the value of money or the rate of surplus value when a theory of prices has been pre-defined. It is shown later that if price determination is simultaneously obtained then one should fix either the value of money or the rate of surplus value. At this moment since a price theory is previously specified, one has the rate of surplus and then following Foley's notion, one has

$$\text{rate of surplus value} \equiv \epsilon = \frac{PY\lambda_m - wLX\lambda_m}{wLX\lambda_m}.$$

Substituting and we can get

$$\epsilon = \frac{LX - w^*LX}{w^*LX} = \frac{(1 - w^*)}{w^*},$$

or one can define a numeraire for such that which means and there is no explicit treatment of the value of money (Lipietz (1980), thus

$$\text{rate of surplus value} \equiv \epsilon = \frac{P'Y - wLX}{wLX}.$$

In both cases one can derive the value of labor power. In Foley's sense this follows from the previous definition, in Dumenil´s case, it is given and defined as

$$w^* = \frac{1}{1 + \epsilon}.$$

Fourth, this approach is not limited to single production but can easily be extended to joint production (see Dumenil (1984a, 1984b).

Fifth, as mentioned before this approach works for value added during the period of analysis rather than gross value.

It does assert that total value added is equal to the total quantity of labor regarded as living labor. Thus, the class distribution of value is a consequence of this total quantity of living labor between capitalists and workers. The relation between the value of labor power and surplus value allows for conflict on its determination.

Six, there is an specific treatment for the notion of intensity and duration of working time so that there is a logical and consistent translation of labor into living labor which can be adopted through this analysis (see Lipietz (1980), appendix) and Foley (1982))

III. THE MODEL OF LIPIETZ

In order to analyze another theory that emphasizes the transformation problem and its relation to the price theory, I introduce Lipietz's model.

The idea behind this solution is based on the specification of a price theory that allows for prices and values to be simultaneously determined meanwhile equalities on total prices and total value and total surplus and total profits is also obtained. A theory of price of production is defined such that a uniform rate of profit is obtained for all sectors.

One can have a 'representative productive process' where living labor adds value to the dead labor incorporated in the means of production and is represented by matrix A. As far as the techniques of production remains constant there is no problem with this representation. Thus, $v = vA + a_n$, where the vector $a = (l_1, \cdots, l_n)$ denotes the vector of quantities of abstract labor incorporated concretely in units of the goods. One then defines as

$$v = a(I - A)^{-1}.$$

In this way, given and α_n, one can derive the values of the system. Furthermore, if one takes the rate of surplus value, $\in$, as given and defines the value of labor power as $w = \frac{1}{1+\epsilon}$, then all one needs is to have given the level of net output y. Thus, we take as given A, α_n, $\in$, and y.

The problem is then defined as follows,

$$\begin{cases} P^* \cdot y = vy \\ P^* = (1+r)(PA + wa_n). \end{cases}$$

If we choose the *numeraire* so that the sum of value added in value terms is equal to the sum of prices of net product, then the sum of profits is equal to the sum of surplus value. Note that in this case, the value of money is one and the numeraire has been chosen so that this equality holds. The vector of reallocated values defines the system of relative prices of production (the level of prices depending on the choice of the numeraire) (Lipietz (1982)).

First, there is the question of existence of a solution to the system, that is, for all y, there exists $(P^*, (1+r))$ that solves the previous two equations. By using Frobenius theorem for non-negative matrix one has the largest eigenvalue for and as far as is smaller than R, P^* can be expanded as a weighted sum

$$P = (1+r)w\sum_{n=0}^{+\infty}((1+r)^n a_n A^n)$$

Then P^* is a continuous increasing function of $(1 + r)$ and P^* is also continuous increasing function of $(1 + r)$. Thus as r approaches the maximum profit rate, P^* tends to infinity. There exists one and only one vector P^* and one and only one value r. The critical value r depends on y.

Consider y and x net and gross output respectively. We have by definition,

$$\begin{cases} P^* \cdot y = v \cdot y \\ x(I - A) = y. \end{cases}$$

In order to show that the sum of profits is equal to the sum of surplus, one writes

$$\begin{aligned} \text{the sum of profits} &\equiv P^* \cdot y - w \cdot a_n \cdot x \\ &= v(I - A)y - wa_n y \\ &= \epsilon w a_n x \\ &= \text{the sum of surplus valı} \end{aligned}$$

In this way, one has shown that by introducing a theory of price with a uniform rate of profit, it has been proved that both conditions hold simultaneously for any level of net output. We have replicated all the results obtained before. This specification of the value of labor power as a share $\frac{1}{1+\epsilon}$ of the value added and also the choice of an appropriate numeraire, so that the sum of value added in value terms is equal to the sum of prices of net product, has allowed us to solve both conditions simultaneously.

IV. THE MODEL OF MORISHIMA

Our next step is to employ a solution that let us contrasts both of these approaches to the solution of Marx transformation problem. On the one hand, one has the theory and solutions already presented by Foley-Dumenil and Lipietz. On the other hand, the tradition developed from Bortkiewicz´s contribution up to Morishima´s and Shaikh´s solution to the problem of transforming values into prices.

In general terms, the idea proposed by Morishima (1974) and this tradition have been to take the value of labor power from a different starting point. A vector describes the worker's consumption bundle. It is stated that $pd = w$ is the variable capital to hold constant through the transformation procedure. Instead of taking $\in$ as given, the rate of surplus value, $pd\ (1 + \in)$=1 is employed and is deduced out of this equation. If represents the technical coefficient matrix we have then that

$$P = (1+r)(PA + wa_n) = (1+r)(PA + Pda_n) = (1+r)P(A + da_n) = (1+r)PA^+,$$

where represents the technical coefficient matrix expanded by the workers consumption basket. One can rewrite the model compactly as

$$P \cdot \frac{1}{(1+r)} = PA^+,$$

and it is evident that as far as Frobenius theorem requirements are satisfied, P and $\frac{1}{(1+r)}$ will be the left eigenvector and the maximum eigenvalue of correspondingly.

Given A, α_n, d, one finds υ, $\in$. Writing

$$\begin{cases} P = (1+r)(PA + wa_n) = (1+r)PA^+ \\ \qquad\qquad Py' = vy'. \end{cases}$$

One has that, in general, the system has a solution such that total value is equal to total price given the choice of the numeraire. The vector is the eigenvector associated to . However, note that in general total surplus value is not equal to total profits unless is chosen to be the von Neumann balanced growth output ray, . That is, unless is the left eigenvector of , in which case, the total surplus value will be equal to total profit,

$$\begin{aligned} \text{Sum of profits} \quad &\equiv \quad rPA^{+}y^{*} = \frac{r}{1+r}Py^{*} = \frac{r}{1+r}vy^{*} \\ &= \quad \frac{r}{1+r}vy^{*} = vy^{*} - vA^{+}y^{*} \\ &\equiv \quad \text{Sum of surplus value,} \end{aligned}$$

if it is the case that $A^{+}y^{*} = \frac{1}{1+r}y^{*}$.

Morishima and Shaikh generated an iterative process to actually derive prices of production from initial values. Shaikh (1977) applies this iterative process and keeps constant the surplus value and variable capital in each step of the iteration. The basic insight for this iterative process derives from the general form $AX = \lambda X$ that the price equation takes. That is, by imposing adequate substitutions and conditions for , particularly being a primitive matrix, one can have a sequence converging toward the equilibrium price.

Given A, α_n, one obtains the value vector υ. Having y_o, a level of gross output, Morishima (1974) derives the following algorithm to obtain the stationary value for y^*.

$$y_t = \frac{vy_{t-1}}{vA^{+}y_{t-1}}A^{+}y_{t-1} \Longrightarrow y^{*}.$$

Thus, having given, one can derive the rate of profit as

$$r = \frac{\epsilon v d a_n y^{*}}{vA^{+}y^{*}}.$$

Finally using the price equation as an iterative process, one obtains the price of production vector insofar as A^{+} is primitive.

$$P_t = P_{t-1}(1+r)A^{+} \Longrightarrow P^{*} = P^{*}(1+r)A^{+}.$$

Again this solution only solves for one of both conditions. However as shown previously if is assumed to be the the von Neumann's balanced growth output ray then both conditions hold,

$$\begin{cases} P^{*}y^{*} = vy^{*} \\ Sy^{*} = \Pi y^{*} = (v - vA^{+})y^{*} = (P^{*} - P^{*}A^{+})y^{*}. \end{cases}$$

Several remarks follow. First, both of these solutions tend toward the same result, that is, the iterative and the maximum eigenvalue strategy for search in the Bortkiewicz-Morishima tradition. Second, the key variable for this analysis is the definition for where this variable represents the workers consumption basket. It is useful to notice the impact of this definition on the augmented technical matrix . From this point, one can show that the rate of profit, , is modified by changes in and not in . Moreover, the workers' consumption basket is held constant through the transformation. Third, there are some problems with this technique to include the case of joint production. Fourth, as has already been noticed there is an explicit theory of price. Fifth, there is always a reference to the gross output rather than the net output. Sixth, the ad hoc determination of implies that the rate of surplus , is not equal to the rate of exploitation derived in the Lipietz's case. In Lipietz's case previously analyzed we had given as a share of the value added. Thus in the Lipietz's case, the value of labor power does not conserve the value of the basket but rather the portion of the total value added.

One can also show what Lipietz means by "the value of commodities recovered by Capitalists is in fact the surplus value. Even if the sum of profits is not as the sum of surplus value, the value of the uses of profits is certainly the sum of surplus value" (Lipietz (1980, 66)). Let us define

$$X = A^{+}X + C + A^{+}\delta X$$

where is gross output, is total input requirement, unproductive consumption for capitalist and the expansion of production. Then we can show the assertion that,

$$\begin{aligned} X &= A^{+}X + C + A^{+}\delta X \\ \Longrightarrow v(X - A^{+}X) &= v(C + A^{+}\delta X) \\ \Longrightarrow \epsilon w a_n X &= v(C + A^{+}\delta X), \end{aligned}$$

Thus, total surplus is equal to the total expenditure of capitalist. This is an interesting implication which can be linked to the divergence of prices from values and also the actual gross output from Neumann's balanced gross output. This implication gives an important insight to theorists of transformation regarding the idea of creation of value in production and conservation in distribution.

It is the case that one can define the distances of and from and respectively where the last two vectors are Neumann's balanced growth prices and output rays. Decompose any actual into a component paralell to the Neumann's ray and a component in the hyperplane orthogonal to the price system , then we have that

$$Y = Y^{*} + Y', \text{ where } \; P^{*} \cdot Y^{*} = vY^{*} \; \text{ and } \; P^{*} \cdot Y' = 0$$

If one writes

$$\begin{aligned} \delta v &= P^{*} - v \equiv \text{divergence of } P^{*} \text{ from } v \\ \delta Y &= Y - Y^{*} \equiv \text{divergence of } Y \text{ from } Y^{*}, \end{aligned}$$

then

$$\delta v \cdot Y + v \cdot \delta Y = 0.$$

In fact,

$$\begin{aligned} &\delta v \cdot Y + (P^{*} - v) \cdot Y = P^{*} \cdot Y^{*} + P^{*} \cdot Y' - v \cdot Y^{*} - v \cdot Y' \\ &\Longrightarrow \delta v \cdot Y = -v \cdot Y' = -v \cdot \delta Y. \end{aligned}$$

Thus, it is shown that in a Morishima's type of model, any 'deformation' from the von Neumann's balanced growth ray has a measure which is equal to "the measure of the difference between total price of production (and total value) for that structure of production" (Lipietz (1982, 68)). Shaikh (1984) has analyzed how this deviation can be justified on his model and how surplus value can deviate from total profits. Basically, surplus value is not being reintroduced in the system when profits are spent as unproductive consumption in non-basic goods.

CONCLUSIONS

Contrary to the idea proposed by Flaschel (1984), Dumenil-Foley solution to the TP is not only a useful method for working out with the degree of freedom of the equation system, but also as shown here, Dumenil-Foley solution to the TP has two further meaningful implications. On the one hand, a re-evaluation of the LTV is proposed that claims that the total value added of the mass of commodities produced is the total quantity of labor expended in producing them. The main concern of the role of the LTV and the classical approach based on distribution theories is turned around. This remark has serious implications in our understanding of prices and values in Marx conception. "Capital" is a book written with a recurrent mixing of price and value and it is difficult to conclude on how much emphasis to pay on individual value formation and aggregate-average value formation. Dumenil (1984a) observes that this is an expository technique employed by Marx and it should not be mixed with Marx's concept of the LTV. Foley-Dumenil's argument is strong in the sense of providing further structure and coherence to the transformation problem to the point of divorcing values from price and eliminating ipso facto the so-called transformation problem.

On the other hand, there is a line of argument that makes difficult to integrate the classical work with the theoretical body of this new approach to Marx Economic Theory. This is derived from the reading of Marx theory of exploitation. As has been point out before, the classical concept of variable capital is seriously damaged with the approach proposed by Dumenil-Foley. The center of the theory of class conflict resides on Ricardo's notion of the basket of consumption for workers and turns around much work developed since Bortkiewicz all the way down to Shaikh's and Morishima's at the end of the 70's. As Dumenil (1984b), remarks on the work of Lipietz there is not much conciliation to be obtained from Morishima type of work and his own. The fact is that once one conceals for either a given rate of surplus as a share of the value added or a given workers consumption basket, then one is "turning the car" into a different direction. It turns out to be difficult to replicate any of the other results obtained from the other approach. The whole project turns around on whether one works with a maximum eigenvalue approach by an augmented technical matrix which reflects the "reduction of the worker to a beast of burden which needs its feed" or brings to the center point the class conflict argument embody in the determination of the rate of surplus. Sraffa (1960) has pointed out for instance an important consideration for this exclusion which is the problem with the basic and nonbasic commodities when one places in workers' share in total value added.

Finally special attention must be placed on the tools developed to understand how to employ the allowed degree of freedom. The question turns toward the study of the determination of these given variables. In the case of Foley, an interesting agenda is to develop a full dynamics of technology to allow for inflation based in cost reduction. Foley (1986) has already advanced part of this inquiry providing the basis for the determination of the value of money. In the case of Lipietz, one further line of research is to have a more explicit investigation in the analysis of the otherwise ad hoc determination of the rate of profit. Dumenil and Levy has part of this worked out with the reconstruction of the financial side in Panico's or Flaschel's sense and the interest rate influencing the rate of profit to close the system. Finally, the study of perturbations to the structure of the system under some specific behavioral rules and its implications to key variables as prices or utilization levels might bring helpful insights concerning the more difficult problem of providing a theory that determines the magnitudes of these defined variables (Foley (1982, 38)). Still, definitional steps must necessarily precede this further task of value determination.

BIBLIOGRAPHY

Dumenil, G., *De la valeur aus prix de production*, Paris, 1980.

Dumenil, G. "Beyond the Transformation Riddle: A Labor theory of value," *Science and Society*, Paris, 1984a.

Dumenil, G. "The So-Called Transformation Problem Revisited: A brief comment," *Journal of Economic Theory*, Vol. 33, 1984b.

Flaschel, P. "The So-Called Transformation Problem Revisited: A brief comment," *Journal of Economic Theory*, Vol. 33, 1984.

Foley, D.K. "The Value of Money, The Value of Labor Power and The Marxian Transformation Problem." *Radical Review of Political Economy*, Vol. 14, No 2, 1982.

Foley, D.K. *Money, Accumulation, and Crisis*, New York, 1986.

Lipietz, A. "The So-Called Transformation Problem Revisited." *Journal of Economic Theory*, Vol. 26, 1982.

Marx, K. *Capital: a critique of political economy*, Vol. I to III, Penguin Books, 1981.

Morishima, M. Marx in The Light of Modern Economic Theory, *Econometrica*, Vol. 42, No 4, 1974.

Shaikh, A. "Marx's Theory of Value and the "transformation Problem" *The Subtle Anatomy of Capitalism,* Vol. 42, No 4, 1977.

Shaikh, A. "The transformation from Marx to Sraffa" *Ricardo, Marx, and Sraffa*, London, 1984.

Sraffa, P. *Production of Commodities by Means of Commodities*. London, 1960.