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Suggestions on How to Combine the Platonic Forms to Overcome the Interpretative Difficulties of the *Parmenides* Dialogue(*)

Abstract: *This paper provides an original approach to research on the logical processes that determine how certain forms participate in others. By introducing the concept of relational participation, the problems of self-referentiality of the Platonic forms can be dealt with more effectively. Applying this to the forms of likeness and unlikeness in Parmenides 132d-133a reveals a possible way to resolve different versions of the Third Man Argument. The method of generating numbers from oddness and evenness may also be of interest; relational participation in these forms clarifies the interpretation of Parmenides 143e-144a.*

Keywords: *Plato. Dialectical method. Participation. Third man. One over many.*

Resumen: *El presente trabajo aporta un enfoque original a la investigación de los procesos lógicos que condicionan que unas formas participen de otras. La introducción del concepto de participación relacional permite gestionar con eficacia los problemas de autorreferencia de las formas platónicas. En su aplicación a lo semejante y lo desemejante en Parménides 132d-133a, aflora una posible vía de solución de las distintas versiones del argumento del «tercer hombre». Puede resultar de interés, asimismo, el método de generación de los números a partir de lo par y lo impar; la participación relacional en estas formas facilita la interpretación de Parménides 143e-144a.*

Palabras clave: *Platón. Método dialéctico. Participación. Tercer hombre. Uno sobre muchos.*

1. Introduction

It is very likely that Plato used a system of symbolic notation when developing the dialectical science. He must have had a simplified way to express combinations of different forms, representing them with letters or other symbols. It is difficult to see how else he could have created the *Parmenides* dialogue, considering its logical complexity. Written in the fullness of his thoughts, it hides his intentions; it ensnares the reader, confusing them with words with multiple meanings; it uses ambiguous expressions and gives rise to incorrect interpretations. Almost twenty-four centuries later, it still resists being interpreted with a sufficiently clear meaning. As there are no reliable data or references, it is difficult to reconstruct the symbols or diagrams he may once have used.¹ Whether or not Plato designed such a system, the logical resources suggested below are of substantial benefit for understanding the dialogue's ambiguous arguments, and they contribute to preserving the consistency of the set during analysis. In this paper, I will test these new tools on different passages, trying to solve some of the problems that have resisted other interpretations.



2. Symbolisation of the central hypotheses of the dialogue

The *Parmenides* discusses eight hypotheses (or eight groups of arguments) related to the concept of the one. Four of these –the first, fourth, sixth and eighth– emphasise the limitations of Parmenidean thought, demonstrating the contradictory results that follow from envisaging a one that has no parts and is not a whole. These arguments are not made solely for critical purposes; they often clarify the assertive use of the dialectical method that is applied to the other four hypotheses.² These remaining hypotheses look at four expressions formed from pairs of very broad opposing forms: *one* (*o*) and *not-one* (\bar{o}), *being* (*b*) and *not-being* (\bar{b}).

In the second hypothesis (H2), the *one* is linked with *being*, forming “the one that is” (*bo*); in the third hypothesis (H3), this combination leads to the *not-one* being linked with *not-being* in “the not-one that is not” ($\bar{b}\bar{o}$). In the fifth hypothesis (H5), the combination of the *one* and *not-being* produces “the one that is not” ($\bar{b}o$); meanwhile, the *not-one* is linked with *being* in “the not-one that is” ($b\bar{o}$), which is the focus of the seventh hypothesis (H7). The mutual dependencies do not end here. Plato allows communication between “the one that is” and “the one that is not”, as well as between “the not-one that is not” and “the not-one that is”, which is also examined in these hypotheses.³

In each of the basic sentences (*bo*, $b\bar{o}$, $\bar{b}o$ and $\bar{b}\bar{o}$), two forms are directly connected. Following the dialectical method, other opposing forms that are present in these groupings and that lead to their division must be identified, creating new formulae.⁴ Among the Greatest Kinds, this role is played by the forms of *sameness* or *identity* (*s*) and *difference* or *contrariety* (\bar{s});⁵ with these, eight combinations are obtained: *sbo*, $s\bar{b}\bar{o}$, $\bar{s}\bar{b}o$, $\bar{s}b\bar{o}$, $s\bar{b}o$, $s\bar{b}\bar{o}$, $\bar{s}b\bar{o}$ and $\bar{s}bo$. To get an idea of the logical values of some of these statements, it is worth taking the proposition for which Plato provides the most information, “the one that is” in H2, and examining the two formulae that are derived from it.

When “the one that is” participates in *same-ness*, this establishes that the *one* and *being* are inseparable; they are interconnected to the extent that the formula *sbo* constitutes a “whole”, which cannot be divided to isolate any individual form:

So what is one is a whole and has a part.

Of course.

What about each of the parts of the one which is, namely, its unity and its being?

Would unity be lacking to the part which is, or being to the part which is one?

No.

So once again, each of the parts contains unity and being, and the least part also turns out to consist of two parts, and the same account is ever true: whatever becomes a part ever contains the two parts. For unity ever contains being, and being unity; so that they are ever necessarily becoming two and are never one.

Quite so.⁶

However, when “the one that is” participates in *difference* instead, i.e. in the sentence $\bar{s}bo$, in this case it is feasible to separate out the multiple “parts” or individual forms that it contains:

Let us see. Since unity is not being, but, as one, gets a share of being, the being of it must be one thing, and it must be another.

Necessarily.

Now, if its being is one thing and unity is another, unity is not different from its being by virtue of being one, nor is its being other than unity by virtue of being; but they are different from each other by virtue of the different and other.

Of course.⁷

Later in this paper, I will explain how Plato regroups these forms to produce evenness, oddness and the numbers. This allows a bijection (to use an anachronistic term) to be established between $\bar{s}bo$ and *sbo*, between the “parts” and the “whole”.⁸

Other attributions of meaning are likewise open for debate; direct participation in *not-being* could refer to thought, supported by words and other representations,⁹ and participation in *being* could refer to whatever is independent of thought,

the “thing itself, which is knowable and truly exists”. The self-predication caused by separating a form (Fness) from its use as a predicate (F)¹⁰ is neatly avoided by introducing a specific form to denote language and predication. This interpretation also circumvents any conceptualist ontology in the strong sense, as it legitimises combinations of forms that do not immediately participate in *thought* or *not-being*.¹¹ There is no longer any danger in acknowledging that the study of forms, structured around the logical formulation of their various valid combinations, leads to ambiguities regarding the linguistic and ontological values of those forms. (Anything that itself participates in *being* and not in *not-being* must still be denoted by a logical symbol that allows to work with groupings of the corresponding forms.)

3. The distinction between immediate participation and relational participation. Application to the forms of *identity* and *contrariety* in *Parmenides* 147b

The concept of “participation” is obscure and controversial, a description that could broadly apply to all of the main technical terms of Platonic doctrine. Throughout this article, it has simply been used to denote the means of communication between forms or groups of forms.¹² It is reasonable to postpone any consideration of the gnoseological and ontological implications until a better understanding of the logical constraints introduced by the dialectical method is obtained.

In the statements sbo , $s\bar{b}\bar{o}$, $s\bar{b}o$, $sbo\bar{}$, $s\bar{b}o\bar{}$, $s\bar{b}o$, $s\bar{b}\bar{o}$ and $s\bar{b}o\bar{}$, the forms are combined without making any distinction between variables and logical operators and without adding any terms between them; this is called “immediate” or “direct” participation. After taking the precaution of restricting these formulae to the lower segments of the intelligible or generated realms (thus preventing any regression to infinity), they can now be recombined with each other through the forms of *identity* and *contrariety* themselves. Two statements can participate,

through each other, in forms that are in turn grouped into higher-level formulae belonging to the higher segments of the intelligible or generated realms; this is called “relational”, “mutual” or “mediate” participation. (Insofar as sbo and $s\bar{b}o$ are inferred relationally, the ranges of the “whole” and the “parts” must be specified for the isomorphism referenced in the previous section.) This concept has implications for the general doctrine of forms, as it prevents direct self-reference and its resulting paradoxes. If a statement participates in a form immediately, it does not participate directly in itself again or in its opposite; however, in relation to another statement, it may participate in the same form (relational reflexivity) or in its opposite (without violating the principle of non-contradiction), as applicable.¹³ This is the distinction underlying statements like the following:

So unity, it seems, is different from the others [$s\bar{b}\bar{o}\bar{}$ \bar{o} - sbo] and itself [$s\bar{b}\bar{o}$ \bar{o} - sbo], and the same as the others [$s\bar{b}\bar{o}$ \bar{o} - sbo] and itself [$s\bar{b}\bar{o}$ \bar{o} - sbo].

Yes, so it appears from this account.¹⁴

I omit the discussion of the reasoning that leads to this conclusion; the reader can confirm its consistency using the truth tables given below. The gaps left open in the meaning of the statement have been filled in with symbols between brackets. Plato avoids defining which “one” he is referring to on each occasion, whether it is “the one that is” or “the one that is not”; he also fails to report its participation in *sameness* or *difference*, and proceeds likewise with “the others” or “the not-one”. This is a deliberate method of playing with the ambiguities of language, omitting the necessary information and producing confusion without causing contradictions. However, specifying the expressions any further would give Plato’s writing a literary tone that would be difficult to evaluate. That is, stylistically speaking, is it not better simply to state that “the one is different from the others” rather than specifying that “the one that is, to the extent of its sameness, is contrary to the not-one that is not where this participates in difference”? Consider

how Plato's full statement appears in a free translation into more rigorous language:

Thus, the one that is, to the extent of its sameness, seemingly opposes the not-one that is not where this participates in difference ($\bar{s}\cdot\bar{s}\bar{b}\bar{o}$ -sbo), whereas by having difference it is contrary to the one that is not in its sameness ($\bar{s}\cdot\bar{s}\bar{b}$ o-sbo); furthermore, the one that is, when it participates in difference, is identical to the not-one that, not being, has sameness (s-sb \bar{o} -sbo), and conversely, when it participates in sameness, it is identical to the one that is not in its difference (s-s \bar{b} o-sbo).

The relational use of participation allows various meanings to be superimposed onto the same form, and Plato uses this skilfully to create confusion in the dialogue.

4. Considerations on dialectical logic. The possibility of creating truth tables adapted for symbolisation

The forms of *identity* and *contrariety* are fundamental threads in the conceptual fabric of the *Parmenides*, constituting the main links between formulae. Before continuing with the dialogue, I should first clarify some points of the logic underlying the dialectics. The procedures used sometimes differ from the standard procedures of logic.

4.1 *Not-being* is a form and not a connector. Opposing forms. Derived formulae and hypotheses with opposing truth values

It is probably the *Sophist* that most clearly states that *not-being* is an individual form:

VISITOR: Because he [Parmenides] says, I think, 'For never shall this prevail, that things that are not are; / I tell you, keep back your thought from this path of inquiry.'

THEAETETUS: Yes, he does say that.

VISITOR: Whereas we have not only shown that what is not is, but have declared what the form of what is not actually is; for having shown up the nature of difference as something that is, cut up into pieces over all the things that are in their relationships with each other, we took our courage in our hands and said of the part of it that is contraposed to the what is of each thing that it was the very thing that what is not really is.

THEAETETUS: Yes, stranger, and what we have said seems to be as completely true as it could be.¹⁵

The reasoning that leads to this conclusion confirms that *being* and *not-being* are opposites in the lower segments, by dividing the higher-range formula that combines *difference* with *being*. (I suggest rereading *Sophist* 257b-258e from this perspective.) The fact that Plato started from the latter two forms to demonstrate the existence of *not-being* does not imply that the same division cannot be achieved by grouping *identity* with *being*. There is also no reason to infer that *not-being* cannot participate immediately in *sameness*. It is more reasonable to suppose that, relationally, *not-being* is subordinate to *being*.¹⁶

It must never be assumed that *not-being* is the "opposite" of *being* in the sense that this term has in the simpler logic of statements. As *not-being* is an individual form, it cannot be treated as a propositional formula derived by applying the negative operator to *being*; its significance is therefore not completely restricted.¹⁷ The type of opposition that arises between *not-being* and *being* may very well reflect the distinction between whatever exists as *thought* (or *language*) and whatever is separately from this.

Despite what the symbols used above may suggest, no isolated Platonic form can be equated to a propositional variable with two or more truth values in the calculus of statements. This is an erroneous formalisation of the language of the *Parmenides* that has been damaging for the various approaches to this work undertaken using the tools of logic. Different truth values must be attributed to combinations of forms.

Both *not-being* and *being* must be combined with other forms to produce sentences that occasionally have opposing truth values. (In the interpretation, the opposing formulae may not be mutually exclusive in an absolute sense; as such, it would be logically viable to say something false about something that exists or to express a truthful opinion about something that happens or is in motion.)

In Platonic dialectics, if two forms are “opposites” (*being* and *not-being*, *one* and *not-one*, *sameness* and *difference*, etc.) then they cannot be combined directly in propositions. In some cases, derived statements can mutually or relationally participate in *contrariety* (e.g. $\bar{s}bo$ participates in \bar{s} relative to $\bar{s}\bar{b}o$), and opposing truth values can then be assigned to them, even if the interpretation is not exclusionary. In other cases, with certain hypotheses, the concept of opposition appears to take on a more restrictive meaning. Probably to prevent the emergence of new segments or levels, Plato states – this time in the more conventional, logical sense – that “the one that is not” ($\bar{b}o$) is the complete opposite of “the not-one that is not” ($\bar{b}\bar{o}$):

Then just what is this hypothesis, “if unity is not”? Doesn’t it differ from “if not unity is not”?

Of course.

Does it only differ, or is it in fact completely opposite to say “unity is not” and “not unity is not”?

Completely opposite.¹⁸

Maintaining the symmetry between the basic sentences, “the one that is” (*bo*) and “the not-one that is” ($\bar{b}\bar{o}$) would also be negations of each other. It is very important to understand these opposing hypotheses in order to build the truth tables. However, it must be emphasised that these opposing truth values are restricted to the lower segments of the intelligible and generated realms.

4.2 Justification of the symbols used

It is dangerous to use a standard logical language when examining the arguments of the *Parmenides*, as this could distort the Platonic

conception of the forms. That is what happens, for example, in a conventional logical formalisation of *identity*: does it remain as a form itself, or does it turn into an operator or a logical connector, like the biconditional? Is first-order logic sufficient to identify the role played by a “whole”? Should it be replaced with a universal quantifier? The same should also be said about its opposite, *difference*, which would be equated with exclusive disjunction and cause difficulties with the negation operator; the associated concept of “parts” would also be problematic, leading to the existential quantifier. It is too early to decide on the logical values that should be assigned to the forms within standard logical languages.

I have tried to avoid these difficulties by using a simple and completely intuitive system of notation. Each form is represented by a letter. To indicate opposing forms, the same letter is used with or without an overline. Forms can be combined to produce formulae; these in turn can be combined into new formulae or higher-level relational expressions, which can be recognised in the notation as they are followed by a simple interpunct. When considering them separately, their ranges must be stated explicitly, but no confusion is caused when a formula immediately connects a sufficient number of forms. (The truth tables constructed for these formulae will have to indicate that they belong to different equivalence classes.)

In the Platonic dialogues, letters of the alphabet often appear and are combined into syllables and words, as an example or model to illustrate dialectical procedures.¹⁹ Greek numerical notation did not generally use its own symbols, but used letters of the alphabet instead. Combinations of letters, based on the tables of opposites linked with Pythagoreanism and inspired by systems of numerical representation, may have been devised to denote the essences of things.

The *aporiae* found in the first half of the *Parmenides* can be interpreted as an obscure method of indicating the rules to follow when combining forms (it is easier to define what must be done by starting with what cannot be done). In this sense, recreating the most basic symbolic system

that Plato may hypothetically have used at the time, adapted to the Latin alphabet for greater convenience, is the best strategy for handling the difficulties of the dialogue. Once the rules are understood, it will be possible to investigate whether dialectics is compatible with a logical system or with a theory, understanding both concepts in the strictest logical sense used today.

4.3 Truth tables for formulae belonging to the lower segments of the intelligible and generated realms

The peculiarities of Plato's philosophy do not prevent the use of truth tables (with minimal adaptations) for checking the consistency of this proposed reading of the *Parmenides*. This simply

requires avoiding any treatment of individual forms as propositional variables; being aware that, although combining a form with others may implicitly introduce a certain operator, this does not reduce the form to that operator; and trying not to confound the truth values taken within basic sentences by the formulae derived by partition. Table 1 shows the truth values of the four main statements and the eight derived statements. Contradictory hypotheses were taken into account for its construction. I have also considered the doctrine, broadly expressed in the *Sophist*, that *what is different from being is a part of not-being*, which is used to identify those lower-range formulae that belong to a single equivalence class. Preserving the symmetry, what is *different from not-being* is likewise a part of *being*:²⁰

Table 1
Truth tables for the formulae corresponding to one and not-one in the lower segments of the intelligible and generated realms

bo		$\bar{b}o$		$b\bar{o}$		$\bar{b}\bar{o}$	
$\bar{s}bo$	sbo	$\bar{s}\bar{b}o$	$s\bar{b}o$	$\bar{s}b\bar{o}$	$sb\bar{o}$	$\bar{s}\bar{b}\bar{o}$	$s\bar{b}\bar{o}$
	1	1			0	0	
1			0	0			1
0			1	1			0
	0	0			1	1	

The logical expressions shown in Table 1 correspond to the lower segments of the intelligible and generated realms; it is only in these segments that formulae and hypotheses with opposing truth values can be identified. (In another paper, I provide a relational deduction of the statements corresponding to the upper segments and their truth tables; the criteria for identifying which relationships refer to the intelligible realm and which to the generated realm are also specified.²¹) By assigning truth values to the formulae, an effective tool is obtained that can be applied to the dialectic method, a

procedure that uses division to systematically analyse the relationships of *identity* and *contrariety* that exist between the basic sentences. These truth tables are a valuable tool for dealing with the endless traps and ambiguities that run through the *Parmenides*.

Throughout the entire dialogue, Plato tacitly and carefully follows certain rules that limit the two different types of participation. As previously and partially discussed: no form should be immediately linked with itself or with its opposite. Derived formulae that mutually participate in a form or in a group of forms with a

higher range are subject to a similar restriction. The formulae in the lower segments that immediately participate in *being* communicate with those that participate in *not-being*; both groups must belong to hypotheses that differ but are not opposites in the absolute sense. (Note that Plato always avoids combining formulae that arise from dividing “the one that is” (bo) and “the not-one that is” ($bo\bar{}$) with each other; he also rejects the relational combination of formulae belonging to “the one that is not” ($\bar{b}o$) and “the not-one that is not” ($\bar{b}\bar{o}$).

5. The use of relational and immediate participation in the forms of likeness and unlikeness in *Parmenides* 148c-d. Overcoming the Third Man in relation to likeness (*Parmenides* 132d-133a)

The formulae in Table 1 are well formed; the forms that compose them participate directly in each other, without any form being immediately combined with itself or its opposite. However, it is possible for a statement to participate in one form (or formula) with respect to another. I have shown some examples of this relational mode of participation in the case of *sameness* and *difference*; the linked propositions had an *identical* or *contrary* truth value. Something similar happens with the opposing forms of *likeness* (l) and *unlikeness* (\bar{l}), in which the formulae allow not just immediate but also mutual or relational participation. In H2, *sameness* and *difference* in particular are used to identify the relationships of *likeness* and *unlikeness*. Once again, the deliberate ambiguity of the text and the absence of an explicit formal language serve to confuse the reader:

So since unity is the same as the others, and because it is different, on both grounds or on either it is both like [$ls\bar{l}s\bar{b}o\bar{l}sbo$] and unlike the others [$\bar{l}\bar{s}\cdot ls\bar{b}\bar{o}\bar{l}sbo$].

Of course.

So too in like manner for itself; since it appeared different from itself and the same as itself, on both grounds and on either it will appear like [$ls\bar{l}s\bar{b}o\bar{l}sbo$] and unlike itself [$\bar{l}\bar{s}\cdot ls\bar{b}o\bar{l}sbo$].

Necessarily.²²

The meaning of the text is clarified by the symbols added in brackets. These are the same formulae that were obtained above when considering *identity* and *contrariety*, now adding the forms of *likeness* and *unlikeness*. The need for symbolisation is clear. What is being stated, in a free translation into less ambiguous language, is the following:

Therefore, the one that is in its sameness, and the not-one that is not in its difference, will each by itself have likeness through their reciprocal participation in difference ($\bar{s}\cdot ls\bar{b}\bar{o}\bar{l}sbo$). And if difference affects the one that is and sameness affects the not-one that is not, then they will separately be unlike through their mutual participation in the form that is the opposite of difference ($s\bar{l}s\bar{b}o\bar{l}sbo$). But when the one and the not-one are linked as identical things, they shall be mutually alike because of this ($ls\bar{s}\bar{b}\bar{o}\bar{l}sbo$), whereas they shall be relationally unlike when they are grouped together as things that are opposite to or different from each other ($\bar{l}\bar{s}\cdot s\bar{b}o\bar{l}sbo$).

Of course.

Likewise, since the one that is, in its difference, and the one that is not, in its sameness, have been shown to be contrary to each other, will they not each separately have likeness ($\bar{s}\cdot ls\bar{b}o\bar{l}sbo$)? And won't the one that is, in its sameness, and the one that is not, in its difference, be separately unlike through the reciprocal effect of identity ($s\bar{l}s\bar{b}o\bar{l}sbo$)? Also, depending on the other characteristic that must be assigned, i.e. whether they are relationally alike or unlike, won't the one be like itself in cases where the groups are identical ($ls\bar{s}\bar{b}o\bar{l}sbo$) and unlike when they are different ($\bar{l}\bar{s}\cdot s\bar{b}o\bar{l}sbo$)?

Necessarily.

In the lower segments, derived formulae that individually participate in *likeness* are relationally *unlike*, and those that are separately *unlike*

are mutually *alike*. Despite Plato's convoluted method of presenting this idea, intertwining immediate and relational uses of participation in the forms of *sameness* and *difference* and in *likeness* and *unlikeness*, it is a relatively simple response to the aporia considered in *Parmenides* 132d-133a, in the introduction to the dialogue:

[...] these characters stand, as it were, as paradigms fixed in the nature of things, but the others resemble them and are likenesses of them, and this sharing that the others come to have of characters is nothing other than being a resemblance of them.

Then if something resembles the character, he [Parmenides] said, is it possible for that character not to be like what has come to resemble it, just insofar as it has been made like it? Is there any device by which what is like is not like to what is like?

There is not.

But what is like necessarily has a share of one and the same character as what it is like? Yes.

But will not that of which like things have a share so as to be like be the character itself? Certainly.

So it is not possible for anything to be like the character, nor the character like anything else. Otherwise, another character will always make its appearance alongside the character, and should that be like something, a different one again, and continual generation of a new character will never stop, if the character becomes like what has a share of itself.

You're quite right.²³

The form of *likeness* and whatever participates in it share a new form of *likeness*, different from the initial form, by virtue of which they are mutually *alike*. If, in turn, anything participates in this new emergent form of *likeness*, together they will relationally produce another form of *likeness* at a higher level; this process continues indefinitely. This vicious cycle in an infinite loop is one version of the Third Man Argument. Plato counters it by requiring formulae that are mutually *alike* to be separately *unlike*, and vice versa when they are mutually *unlike*. By introducing this rule, he in fact prevents the property of

likeness from being used self-referentially, even relationally, which is the cause of the regression to infinity in the aporia.²⁴

This proposed technical solution will be more relevant when there is a real need to examine more complete higher-range formulae. (If deduced relationally, *sbo·* and *sbō·* will participate immediately in *likeness*, while *ṡ bo·* and *ṡ bō·* will participate in *unlikeness*.) With a better overview of the formal framework, it will be possible to try applying it to everyday situations where like or unlike things are discussed in natural language. At that time, it will be particularly interesting to examine the examples on friendship that are suggested in the *Lysis* dialogue, which are mixed in with the discussion on pre-Socratic principles of likeness.²⁵ A decision will have to be made on whether symbolisation can be used to interpret these examples in a way that helps resolve the apparent contradictions found at the end of that work.

The distinction between relational and immediate participation produces specific aporiae for some of the more general forms; this is what the first part of the *Parmenides* mainly seeks to demonstrate. Strict rules must be set to prevent paradoxes related to self-reference. As with the forms of *likeness* and *unlikeness*, the peculiarities of *being* and *not-being*, *equal* and *unequal*, *large* and *small*, *older* and *younger*, etc., must also be considered, avoiding the predictable objections.

6. Applying the modes of participation to the generation of numbers in *Parmenides* 143e-144a

Due to the scope of the thesis proposed in the previous paragraph, I will leave this matter for future research.²⁶ Instead, I will now look at one final section of the *Parmenides* where the distinction between relational and immediate participation is also functional. Difficult and sometimes ignored, the beginning of H2 looks at the problem of the generation of numbers. When considering "the one that is" in its participation in *difference* (*ṡ bo*), the forms of *being*, the *one* and *difference* are separable and can

be regrouped into a number of odd or even terms.²⁷ Surprisingly, Plato appears to defend the existence of some kind of correspondence between combinations of oddness and evenness and each number:

So there will be even-times even numbers, odd-times odd numbers, even-times odd numbers, and odd-times even numbers.

True.

Then if this is so, do you think there is any number left which must not necessarily be?

None whatever.²⁸

The meanings that are usually attributed to the expressions “even-times even number”, “even-times odd number”, “odd-times even number” and “odd-times odd number” follow the definitions given in Book VII of Euclid’s *Elements*: Definition 8 states that “an even-times even number is that which is measured by an even number according to an even number”.²⁹ These are numbers that are products of two even numbers, i.e. the series 4, 8, 12, 16, etc. According to Definition 9, “an even-times odd number is that which is measured by an even number according to an odd number”.³⁰ In principle, this would include all products of an even number and an odd number; however, Book IX Proposition 33 specifies that the number is “even-times odd only” (and not “even-times even also”) if it is the double of an odd number, thus producing the series 6, 10, 14, 18, etc. The definition of “odd-times even number” is believed to have been a later interpolation from the *Elements* and is not used anywhere in that work; it is usually considered to indicate a certain type of even number.³¹ If that is so, then Definition 10 is problematic: “an odd-times odd number is that which is measured by an odd number according to an odd number”.³² It is believed that Euclid is referring here to the product of two odd numbers

or, to put it another way, to composite odd numbers. This definition, which is also not found in the *Elements*, cannot be used to complete the set of all numbers because it does not include the prime numbers. This is clearly a problem because, as shown in the previous quotation from the *Parmenides*, Plato states that *once the different types of odd and even number are known, there is no number left which must not necessarily be*.

It may be misguided to interpret Plato’s meaning based on an understanding of similar expressions in Euclid’s work, which was written later in time. In Plato’s work, *odd* and *even* are opposing forms that therefore cannot be combined directly; they also indicate the objects that are covered by these forms, the numbers, which can be generated from each other.³³ It could be argued that “even” means “half of all numbers” and “odd” means “the other half of all numbers”, thus including every number. However, the real difficulty lies in unambiguously determining each number starting from the *odd* and *even* forms, using the possibilities offered by relational participation in the same form or in its opposite. How can this be done?

If the even half of all numbers is taken and the first of these is separated out, the remaining even numbers can then be regrouped, separating them into an “even-even” series and an “even-odd” series; only their odd or even positions with respect to the first even number need to be considered. The same rule of division is then applied to each branch obtained. After the first term of the “even-even” series, the remaining terms of this series are regrouped into “even-even-even” or “even-even-odd”, and so on. The same procedure is used for the odd numbers: after the first, the remaining odd numbers are arranged into odd or even positions, and each of them creates new odd or even positions counting from there. Table 2 shows the first few series:

Table 2
Numerical series for the first even and odd species

<i>Even</i> (2, 4, 6, ...)	<i>Even-even</i> (4, 8, 12, ...)	<i>Even-even-even</i> (8, 16, 24, ...)
		<i>Even-even-odd</i> (12, 20, 28, ...)
	<i>Even-odd</i> (6, 10, 14, ...)	<i>Even-odd-even</i> (10, 18, 26, ...)
		<i>Even-odd-odd</i> (14, 22, 30, ...)
<i>Odd</i> (3, 5, 7, ...)	<i>Odd-even</i> (5, 9, 13, ...)	<i>Odd-even-even</i> (9, 17, 25, ...)
		<i>Odd-even-odd</i> (13, 21, 29, ...)
	<i>Odd-odd</i> (7, 11, 15, ...)	<i>Odd-odd-even</i> (11, 19, 27, ...)
		<i>Odd-odd-odd</i> (15, 23, 31, ...)

Looking at Table 2, it is easy to identify the first term of each series. If 2 is taken as the first “even” number, “even-even” indicates the number 4, “even-odd” indicates 6, “even-even-even” indicates 8, etc.; if 3 is the first odd number, then “odd-even” indicates the number 5, “odd-odd” indicates 7, “odd-even-even” indicates 9, etc. Therefore, the numbers participate in the forms of *odd* or *even* both directly and in their mutual relationships.³⁴ This explanation makes it significantly easier to use a formal treatment, in terms of both set theory and mereology, of the Platonic concept of number: the quantity of series is the same as the quantity of natural numbers. The recursive division of the even numbers into odd and even series, and likewise for odd numbers, allows a one-to-one correspondence to be defined between the first term of each series and the natural number that immediately precedes it.³⁵ It should also be recalled that the elements into which *sbo* is divided follow the pattern 2^n (where n is a non-zero natural number) and constitute a countably infinite set. A bijection can thus be established between the terms of $\bar{s}bo$ and those of *sbo*, between the “parts” and the “whole”.

While it is trivial to prove this with set theory, it is less clear when trying to prove it using the resources that may have been available to Plato. (It seems anachronistic to assume that this was his intention in *Parmenides* 142d-145a.)

The interpretation described above allows the generation of all of the numbers, including those that are not covered by Euclid’s definitions, without having to introduce assumptions from outside of Plato’s philosophy. In particular, it obviates the need to rely on distant sources of Platonism such as Theon of Smyrna, who sees one as an odd number and considers “odd-odd” numbers to refer to the primes (“one times 5 is 5, one times 7 is 7, and one times 11 is 11”).³⁶ It is difficult to fit this explanation in with Plato’s words, and it contradicts his habit of treating the one as the beginning or part of the number and identifying this number with multiplicity.

There are many mathematical questions that would be interesting to consider in the light of a new reading of the *Parmenides*. Attempts to tackle the paradoxes related to set theory have led to the development of different concepts of class. The Third Man Argument is very similar to some

of these paradoxes. Plato's strategy for confronting it, by setting strict rules for the immediate and relational uses of participation, deserves more attention. (At the same time, the distinction between the "whole" and the "parts" would have to be looked at in relation to the concept of the set.) Among other things, it would also be worth exploring the problem of incommensurable magnitudes and the mathematical infinite from the perspective of H7 and its considerations of the latter: by analysing the logical relationships of the "not-one", Plato argues that some pluralities have no true unity or number. The mathematical significance of this statement is made clearer by recalling that this hypothesis discusses certain "masses unlimited in multitude", which appear to have unity and to combine with each other depending on the number, and even to limit each other despite not having limits.³⁷

7. Conclusions

The logical tools developed in this article contribute to a more comprehensive reading of the framework of hypotheses in the *Parmenides*. Their potential and efficacy have been tested on different passages in response to problems that other interpretations have been unable to resolve. There are good arguments in their favour, particularly considering Plato's premeditated effort to avoid providing any unequivocal textual evidence that could be used to assign a clear meaning to his arguments or to the dialogue in general; this issue has not always been given its due value, despite being the root cause of the lack of agreement in the specialist literature.³⁸ A detailed philological examination of the texts is not sufficient to untangle the knots of language that are intended to affect and confuse the reader's powers of reason. The exercise on the dialectic method that Plato provides as a model must be technically reconstructed before moving on to other questions. In that sense, this logical device opens up a wide range of possibilities and is essential for understanding the uncountable arguments that make up the *Parmenides*, particularly if the consistency of the set is assumed.

Notes

- * This article was originally published in Spanish, with some changes, in the journal *Éndoxa*. Series Filosóficas (2019, (43), 41-66, doi: <https://doi.org/10.5944/endoxa.43.2019.22385>), which is published by the School of Philosophy of the Spanish National Distance Education University under a Creative Commons Attribution-Non-Commercial-NoDerivatives 4.0 licence. Used by permission. Translated into English by Katy Robinson.
- Books VI and VII of the *Republic* seem to indicate that the use of "images" makes mathematical knowledge less valuable than dialectics; however, it would be excessive to reject systems of symbolic notation and diagrams that work with the forms themselves for that reason. To better understand Plato's caution, consider his method for working with physical magnitudes. For example, when studying time, he is careful to relate its quantity with the intelligible realm and to identify representations of the past and future with movements in the generated realm (see *Parmenides* 151e-155c and *Timaeus* 37e-38a), and when dealing with the magnitude of a movement, he separates the numerical calculation of distances and the position or place occupied by sensibles into different domains. This method allows velocities to be determined without obstruction, making it easier to respond to paradoxes such as Zeno's arguments against motion. (See Matía Cubillo (c)).
 - See *Parmenides* 142b-157b (second), 157b-159b (third), 160b-163b (fifth) y 164b-165e (seventh). This classification of the hypotheses based on two incompatible interpretations of the one is found in Cornford (1939, 109-115). Turnbull (1998, 47-50) speaks even more explicitly of a "Parmenidean Version" and a "Platonic Version" when classifying the eight parts of the dialectical exercise.
 - See *Parmenides* 136a-c.
 - See *Philebus* 16c-19b.
 - See *Sophist* 254d-255e.
 - Parmenides* 142d-143a, translated by Allen (1997). For this and other direct quotes from Plato, I have transcribed the Greek text according to *Burnet's edition* (1900-1907): —Καὶ ὅλον ἄρα ἐστὶ, ὃ ἂν ἐν ἧ, καὶ μόνιον ἔχει. —Πάνυ γε. —Τί οὖν; τῶν μορίων ἐκότερον τούτων τοῦ ἐνὸς ὄντος, τό τε ἐν καὶ τὸ ὄν, ἄρα ἀπολείπεσθον ἢ τὸ ἐν τοῦ εἶναι μορίου ἢ τὸ ὄν τοῦ ἐνὸς μορίου;

—Οὐκ ἂν εἶη. —Πάλιν ἄρα καὶ τῶν μορίων ἐκότερον τὸ τε ἐν ἴσχει καὶ τὸ ὄν, καὶ γίγνεται τὸ ἐλάχιστον ἐκ δυοῖν αὐτῶν μορίων τὸ μόριον, καὶ κατὰ τὸν αὐτὸν λόγον οὕτως αἰεὶ, ὅτιπερ ἂν μόριον γένηται, τούτῳ τὸ μόριον αἰεὶ ἴσχει: τὸ τε γὰρ ἐν τὸ ὄν αἰεὶ ἴσχει καὶ τὸ ὄν τὸ ἐν: ὥστε ἀνάγκη δὴ αἰεὶ γιγνόμενον μηδέποτε ἐν εἶναι. —Παντάσῃ μὲν οὖν.

7. *Parmenides* 143b, trans. cit. Burnet (1900-1907): —Ἰδωμεν δὴ: ἄλλο τι ἕτερον μὲν ἀνάγκη τὴν οὐσίαν αὐτοῦ εἶναι, ἕτερον δὲ αὐτό, εἴπερ μὴ οὐσία τὸ ἐν, ἀλλ' ὡς ἐν οὐσίας μετέσχευ. —Ἀνάγκη. —Οὐκοῦν εἰ ἕτερον μὲν ἢ οὐσία, ἕτερον δὲ τὸ ἐν, οὔτε τῷ ἐν τὸ ἐν τῆς οὐσίας ἕτερον οὔτε τῷ οὐσία εἶναι ἢ οὐσία τοῦ ἐνὸς ἄλλο, ἀλλὰ τῷ ἑτέρῳ τε καὶ ἄλλῳ ἕτερα ἀλλήλων. —Πάνυ μὲν οὖν.
8. A discussion of the concepts of “parts” and “whole” in the two referenced passages can be found in the monograph by Harte (2002, 78-83). While sympathising with mereology, the author takes a broader perspective when studying these two concepts. However, she does not identify the expression “parts” with a combination of *difference*, *being* and the *one*, but only with each form separately. In her eagerness to emphasise the dependency of the “parts” upon the “whole”, she does not consider *difference* and *identity* as connections to alternative formulae. In other contexts, the two terms do take on the meaning indicated by Harte, distinguishing the formulae of the lower segments from those of the higher segments, in the generated and intelligible realms.
9. It is perplexing that the innovative idea suggested by Marcos de Pinotti (1997, 62, 69, 76, 80) has not been explored from a logical perspective.
10. Cf. Vlastos (1995, 167, 170, 174 n. 13, 180-182).
11. In *Parmenides* 132b-c, Plato mentions the aporiae that result from conceptualism, which states that forms are only thoughts. This means, among other things, that under this assumption each form is a thought of another thought-form, in an implicit regression to infinity. As I have tried to show elsewhere (see Matía Cubillo (a)), preventing formulae that belong to the higher segments of the intelligible or generated realms from participating directly in the form of *not-being* counters this version of the Third Man Argument and gives *being* ontological priority over *not-being*. Plato introduces the rule to follow when creating formulae in the guise of an aporia. (Note that Helming (2007, 323 n. 58) has argued, against authors such as Rickless (2007, 75-80), that the

referenced text does not involve any regression to infinity, and therefore he does not acknowledge any “third man”.)

12. See *Parmenides* 133c-d.
13. By distinguishing these two types of participation, it is possible to deal with controversies regarding the axioms or principles of *self-participation* and *purity* (by which a form cannot participate in its opposite). Self-participation cannot be applied to the forms immediately or directly, but does apply to some of them relationally. The axiom of purity is always valid for immediate participation and in some cases for relational participation. The principal of self-participation has been discussed in the specialist literature. The desire to resolve the contradictions that arise from its inclusion in Plato’s philosophy is clearly evident in Vlastos (1981, 335-365), especially with regard to the Third Man Argument. A good overview of the most controversial issues in the criticism of the *Parmenides* was written by Rickless (Spring 2020).
The concept of mediate or relational participation could also reopen certain parts of an old debate. It would be bold to claim that this tool is essential for the study of Plato, but in any case, its usage certainly challenges the methods used in the referenced studies to approach difficulties with interpreting the dialogue.
14. *Parmenides* 147b, trans. cit. Burnet (1900-1907): —Τὸ ἐν ἄρα, ὡς εἴκειν, ἕτερόν τε τῶν ἄλλων ἐστὶν καὶ ἑαυτοῦ καὶ ταῦτ’ ἐκείνοις τε καὶ ἑαυτῷ. —κινδυνεύει φαίνεσθαι ἐκ γε τοῦ λόγου.
15. *Sophist* 258d-e, translated by C. Rowe (2015). Burnet (1900-1907): —Ὅτι ὁ μὲν πού φησιν— Οὐ γὰρ μήποτε τοῦτο δαμῆ, εἶναι μὴ ἐόντα, ἀλλὰ σὺ τῆσδ’ ἀφ’ ὁδοῦ διζήσιος εἶργε νόημα. —Λέγει γὰρ οὖν οὕτως. —Ἡμεῖς δὲ γε οὐ μόνον τὰ μὴ ὄντα ὡς ἔστιν ἀπεδείξαμεν, ἀλλὰ καὶ τὸ εἶδος ὁ τυγχάνει ὄν τοῦ μὴ ὄντος ἀπεφηνάμεθα: τὴν γὰρ θατέρου φύσιν ἀποδείξαντες οὐσάν τε καὶ κατακεκεραμισμένην ἐπὶ πάντα τὰ ὄντα πρὸς ἄλληλα, τὸ πρὸς τὸ ὄν ἕκαστον μόριον αὐτῆς ἀντιτιθέμενον ἐτολήσαμεν εἰπεῖν ὡς αὐτὸ τοῦτ’ ἐστὶν ὄντως τὸ μὴ ὄν. —Καὶ παντάσῃ γε, ὃ ξένη, ἀληθέστατά μοι δοκοῦμεν εἰρηκέναι. —Ἡμεῖς δὲ γε οὐ μόνον τὰ μὴ ὄντα ὡς ἔστιν ἀπεδείξαμεν, ἀλλὰ καὶ τὸ εἶδος ὁ τυγχάνει ὄν τοῦ μὴ ὄντος ἀπεφηνάμεθα.
16. See Matía Cubillo (a).
17. Fine (1993, 108-110, 112-116, 113 n. 53, 114 n. 55) distinguishes “opposites” from “negations”, considering the former to be forms or properties

- (“genuine features of reality”), while the latter are only “complements of properties or kinds”. Fine admits that, for Plato, *not-being* is a form in its own sense, but as long as no function can be found to isolate it, she accepts the extended interpretation that equates it to *difference* as one and the same form.
18. *Parmenides* 160b-c, trans. cit. Burnet (1900-1907): —Τίς οὖν ἂν εἴη αὐτῆ ἢ ὑπόθεσις, εἰ ἔν μὴ ἔστιν; ἄρα τι διαφέρει τήσδε, εἰ μὴ ἔν μὴ ἔστιν; —Διαφέρει μέντοι. —Διαφέρει μόνον, ἢ καὶ πᾶν τοῦναντίον ἐστὶν εἰπεῖν εἰ μὴ ἔν μὴ ἔστι τοῦ εἰ ἔν μὴ ἔστιν; —Πᾶν τοῦναντίον.
 19. See, among others, *Theaetetus* 203a-204a, *Sophist* 252e-253a, *Statesman* 277e-278e and *Philebus* 17b. Cf. *Metaphysics* 1043b-1044a and 1045a-b.
 20. See *Sophist* 256d-259d. Towards the end of the passage, Plato specifies that *in some sort of fashion a thing is the same when it is different and different when it is the same, where one of the two is affected by what is said*. It is fitting to equate *not-being* with “what is said in words”. (The table of values offered in the original Spanish version of the article has been revised.)
 21. See Matía Cubillo (a).
 22. *Parmenides* 148c-d, trans. cit. Burnet (1900-1907): —Ταῦτόν τε ἄρα ὄν τὸ ἐν τοῖς ἄλλοις καὶ ὅτι ἕτερόν ἐστι, κατ’ ἀμφοτέρα καὶ κατὰ ἐκότερον, ὁμοίον τε ἂν εἴη καὶ ἀνόμιον τοῖς ἄλλοις. —Πάνυ γε. —Οὐκοῦν καὶ ἑαυτῷ ὡσαύτως, ἐπεὶ περ ἕτερόν τε ἑαυτοῦ καὶ ταῦτόν ἑαυτῷ ἐφάνη, κατ’ ἀμφοτέρα καὶ κατὰ ἐκότερον ὁμοίον τε καὶ ἀνόμιον φανήσεται; —Ἀνάγκη.
 23. *Parmenides* 132d-133a, trans. cit. Burnet (1900-1907): —[...] τὰ μὲν εἶδη ταῦτα ὡς περ παραδείγματα ἐστάναι ἐν τῇ φύσει, τὰ δὲ ἄλλα τούτοις εὐκέναι καὶ εἶναι ὁμοιώματα, καὶ ἢ μέθεξις αὐτῆ τοῖς ἄλλοις γίγνεσθαι τῶν εἰδῶν οὐκ ἄλλη τις ἢ εἰκασθῆναι αὐτοῖς. —Εἰ οὖν τι, ἔφη, εἰκεν τῷ εἶδει, οἷόν τε ἐκεῖνο τὸ εἶδος μὴ ὁμοιον εἶναι τῷ εἰκασθέντι, καθ’ ὅσον αὐτῷ ἀφωμοιώθη; ἢ ἔστι τις μηχανὴ τὸ ὁμοιον μὴ ὁμοίῳ ὁμοιον εἶναι; —Οὐκ ἔστι. —Τὸ δὲ ὁμοιον τῷ ὁμοίῳ ἄρ’ οὐ μεγάλη ἀνάγκη ἐνὸς τοῦ αὐτοῦ [εἶδους] μετέχειν; —Ἀνάγκη. —Οὐ δ’ ἂν τὰ ὁμοια μετέχοντα ὁμοια ἦ, οὐκ ἐκεῖνο ἔσται αὐτὸ τὸ εἶδος; —Παντάσῃ μὲν οὖν. —Οὐκ ἄρα οἷόν τέ τι τῷ εἶδει ὁμοιον εἶναι, οὐδὲ τὸ εἶδος ἄλλω εἰ δὲ μή, παρὰ τὸ εἶδος αἰεὶ ἄλλο ἀναφανήσεται εἶδος, καὶ ἂν ἐκεῖνο τῷ ὁμοιον ἦ, ἕτερον αὖ, καὶ οὐδέποτε παύσεται αἰεὶ καινὸν εἶδος γιγνόμενον, ἐὰν τὸ εἶδος τῷ ἑαυτοῦ μετέχοντι ὁμοιον γίγηται. —Ἀληθέστατα λέγεις.
 24. This argument can be developed by identifying the well-formed expressions, starting from each individual form and the types of participation. Language and predication are relegated to *not-being* and cease to obstruct the reasoning. What is *said* and what *is* mutually participate in a formula at a higher range where truth and falsity, or their appearance, are decided (see Matía Cubillo (a)). Therefore, when dealing with this aporia, there has been no need to differentiate the form (F-ness or Φ_F) from its use as a predicate (F). In his classic study of the Third Man, Vlastos (1995, 167 n. 5, 183 n. 39) considered this distinction to be logically and ontologically essential for understanding the aporia. More recent opinions, such as those of Pelletier and Zalta (2000, 167, 181-185), maintain this separation in the logical notation to prevent paradoxes like those of Russell.
 25. Cf. *Lysis* 213d-216b.
 26. See Matía Cubillo (a; b; c).
 27. See *Parmenides* 143a-d.
 28. *Parmenides* 143e-144a, trans. cit. Burnet (1900-1907): —Ἄρτιά τε ἄρα ἀρτιάκις ἂν εἴη καὶ περιττὰ περιττάκις καὶ ἄρτια περιττάκις καὶ περιττὰ ἀρτιάκις. —Ἔστιν οὕτω. —Εἰ οὖν ταῦτα οὕτως ἔχει, οἷε τινα ἀριθμὸν ὑπολείπεσθαι ὄν οὐκ ἀνάγκη εἶναι; —Οὐδαμῶς γε. Long-held tradition, going back to Aristotle (see e.g. *Metaphysics* 987b), relates this passage to the generation of numbers. Allen (1997, 265-267) has argued that Plato is not discussing the formation of numbers, but rather demonstrating their existence and classification (excluding the prime numbers); his arguments are largely philological. Turnbull’s contribution (1998, 73-79) to the problem should also be mentioned. Turnbull uses what he calls the “three machine”, which can be equated to the formula $\bar{s}bo$, to obtain the sequence of dyads or pairs 3, 9, 27, etc.; he also requires the “two machine”, which can be equated to the formula sbo , to produce the progression of simple terms 2, 4, 8, etc. By using different combinations of these two “machines”, he is able to construct the remaining numbers as dyads. His solution can be adapted to involve the opposing forms of *identity* and *contrariety* in the generation of numbers, but it cannot be used to determine them unambiguously. The same thing can be seen in a paper by Scolnicov (2003, 105-106), who suggests obtaining the prime numbers larger than three by subtraction from even numbers.

29. *Elements* VII def. 8, translation by Heath (1908). (Ἀρτιάκις ἄρτιος ἀριθμός ἐστιν ὁ ὑπὸ ἀρτίου ἀριθμοῦ μετρούμενος κατὰ ἄρτιον ἀριθμόν).
30. *Elements* VII def. 9, trans. cit. (Ἀρτιάκις δὲ περισσός ἐστιν ὁ ὑπὸ ἀρτίου ἀριθμοῦ μετρούμενος κατὰ περισσὸν ἀριθμόν).
31. See Heath (1908, 282-284).
32. *Elements* VII def. 10, trans. cit. (Περισσάκις δὲ περισσός ἀριθμός ἐστιν ὁ ὑπὸ περισσοῦ ἀριθμοῦ μετρούμενος κατὰ περισσὸν ἀριθμόν).
33. See *Phaedo* 103a ff.
34. Cf. *Metaphysics* 1004b.
35. The referenced series have the same cardinality: they are countably infinite sets. For even numbers, each natural number can be made to correspond to its double, and vice versa, with each even number corresponding to half its value. In the “even-even” series, the bijection is established between the natural numbers and four times their value; a similar bijection is possible for all other series. At this point, I cannot help recalling D. Hilbert’s *Infinite Hotel*; no matter how complicated it is to prove, the set of all terms in all of the series, however often they are repeated, is also a countably infinite set.
36. Theon of Smyrna (1979, 15).
37. The *Parmenides* dialogue can corroborate some of Alonso Álvarez’ theories (2012, 50-58) on real numbers. Repeating the operation that leads to calculating a number with infinite decimal places produces a series of open intervals that grow endlessly narrower; if the unity they share is dispensed with as being somehow misleading (which appears to follow from the reasoning of the *Parmenides* in H7), the number disappears.
38. See Rickless (Spring 2020).
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- . (b) El *Parménides* y la versión del argumento del «tercer hombre» relativa a la grandeza [The *Parmenides* dialogue and the version of the Third Man Argument regarding largeness]. Pending evaluation.
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