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Why Mathematical Philosophy?[1]

Mathematical philosophy, or, synonymously, *formal philosophy*, is the application of logical, mathematical, and computational methods —in short: *formal methods*— to philosophical questions and problems. Thus, mathematical philosophy in this present-day sense of the term is not the same as philosophy *of mathematics*, even though large parts of philosophy of mathematics do employ formal methods and thus belong to mathematical philosophy as well.[2]

The idea of using formal methods in philosophy is of course not new:

> The only way to rectify our reasonings is to make them as tangible as those of the Mathematicians, so that we can find our error at a glance, and when there are disputes among persons, we can simply say: Let us calculate [*calculemus*]. (Leibniz 1685)[3]

However, it is probably right to say that formal methods have never been as prominent in philosophy as they are right now, and the breadth and diversity of formal methods used in philosophy has never been greater.

But why is it that mathematics can be applied in philosophy at all? The answer is simple: whenever philosophy gets developed in a sufficiently clear and systematic manner, it exhibits *formal structure*; and modern mathematics is nothing but the *study of formal structure*. So it should not be particularly surprising that formal methods may support philosophical work by supplying insight into the very formal structures that philosophical topics, questions, problems, concepts, claims, theories, arguments, and examples instantiate.

Here is an (incomplete) list of ways in which formal methods can facilitate philosophical work and may sometimes even be necessary for it to progress:

- Formal methods can help philosophers to explicate philosophical concepts.[4]

That is: with the help of formal methods, philosophers can clarify, precisify, and refine concepts that are central to their philosophical work —concepts, such as *validity*, *truth*, *meaning*, *knowledge*, *rationality*, *induction*, *existence*, *identity*, *relation*, *necessity*, *obligation*, *value*,, A prototypical example would be A. Tarski's (1935) seminal semantic work on the concept of truth in which he demonstrated how one could state a formally correct and materially adequate explicit definition of truth for a great variety of formalized object languages (such as, e.g., the language of arithmetic, the language of chemistry, or the language of the metaphysical theory of mereology). The formal methods required for that purpose were those of higher-order logic or set theory, one of the benefits of Tarski's definition was that it avoided the occurrence of semantic paradoxes, and his overall approach

became the background of virtually all modern philosophical work on truth.

• Formal methods can help philosophers to systematize and justify philosophical claims and theories.

Much as physicists use mathematical methods[5] to deduce predictions from law hypotheses, empirical data, and auxiliary assumptions, philosophers can employ formal methods to extract philosophical conclusions from philosophical axioms, philosophical case descriptions, and philosophical assumptions:

Philosophical Claim 1.
Philosophical Claim 2.

(Logic and Mathematics.)
Therefore: Philosophical Conclusion.

For instance, the epistemological thesis that *ideally rational subjects must distribute their degrees of belief over propositions according to the laws of probability* can be justified by deriving it from a system of philosophical principles concerning practical or epistemic rationality, combined with the calculus of real numbers and probability theory. In the relevant literature, these justifications are referred to as 'Dutch book arguments', 'decision-theoretic representation theorems', and 'arguments from minimizing inaccuracy' (Hájek 2009; Joyce 1999; Pettigrew 2016), and the mathematics required for them is not different from what is used, say, in statistical mechanics.

• Formal methods can help philosophers to prove the absurdity or even inconsistency of philosophical claims and theories.

One cannot just use formal methods to support one's philosophical theses but one may also apply them to *rule out* certain combinations of philosophical principles by proving these principles to be committed to absurd or outright contradictory consequences:

Philosophical Claim 1.
Philosophical Claim 2.

(Logic and Mathematics.)
Therefore: Absurdity/Contradiction.

The corresponding logically valid arguments from *prima facie* plausible philosophical premises to absurd conclusions are often called 'paradoxes' or, if their conclusions are logically contradictory, 'antinomies'. A paradigmatic example in epistemology would be Fitch's Paradox in which seemingly unproblematic assumptions of typical antirealist positions concerning knowledge and truth (such as that all truths are knowable) are shown to lead to absurd consequences (such as that every truth is known).[6] Paradoxes and antinomies are often the starting points for further philosophical debate in which different ways of avoiding their problematic conclusions are being explored: e.g., one may wonder which of the premises of Fitch's paradox needs to be given up or revised, or one may question whether the system of classical epistemic logic that is required to derive its absurd conclusion is acceptable from an antirealist position. And that leads to philosophical progress.[7]

• Formal methods can help philosophers to prove the consistency of philosophical theories by building models in the logical sense of model theory.

What if a philosopher wants to demonstrate that their philosophical theory does *not* imply a contradiction? They could determine a mathematical model for their theory, much as mathematicians once proved non-Euclidean geometry consistent by constructing models for it. For example, assume you are a metaphysician interested in the ontology of abstract entities who wants to maintain that: (i) for every expressible condition on properties, there is an abstract individual that has just the properties meeting the condition; and (ii) for every expressible condition on individuals, there is a property or relation which applies just to the individuals meeting the condition. Is it consistent to maintain (i) and (ii) simultaneously? Well, the answer depends on the

details, of course, such as what 'every', 'expressible condition', 'property/relation', 'there is', 'abstract', 'individual', 'has', 'meet', 'applies to', and 'just' mean in the two theses. But once the details have been provided in sufficiently clear and precise form, the resulting theory of abstract entities might well be proven consistent by supplying a mathematical model for it. In fact, this is not just a fictional story: E. Zalta developed an axiomatic theory precisely like that[8], logicians constructed models for it (such as D. Scott and P. Aczel), and one can even use automated theorem provers to check for its consistency (cf. Kirchner 2017).

- Formal methods can help philosophers to argue for the plausibility or implausibility of philosophical claims and theories by enabling them to build and study models in the scientist's sense of the term.

Scientists often investigate empirical phenomena by building and studying models of these phenomena, but not logical models as discussed before but rather systems of idealized mathematical constraints and assumptions that partially and approximately represent the phenomena in question (such as the Lotka-Volterra system of differential equations which represents the interaction between predators and prey in some given population). In an analogous manner, philosophers can investigate philosophical topics by building and studying «quasi-scientific» models of them: e.g., Brian Skyrms has been using models and methods from evolutionary game theory to successfully investigate topics in social philosophy (cooperation, fairness, convention,) (cf. Skyrms 2014). Philosophical models in that sense do not only allow for the application of computer simulations but may sometimes even necessitate their use because the mathematics might get too complicated otherwise. Indeed, it has become more and more common in some areas of philosophy (formal epistemology, general philosophy of science,) to argue for the plausibility or implausibility of philosophical theses by determining whether they hold true in simulated models.

In a nutshell: *formal methods can do a lot of good in philosophy*. Of course, this does not mean that they will always pay off, and even when they do, it usually requires significant and sometimes ingenious preparatory work by which they become applicable to philosophical questions and problems in the first place. But more often than not it is worth the effort, from which I conclude that one part of the philosophy of the future (next to others) will be mathematical. *Calculemus!* [9]

Notas

1. I would like to express my gratitude to Lorenzo Boccafogli for urging me to write this little note for the *Revista*.

2. For instance, B. Russell's *Introduction to Mathematical Philosophy* (1919) is really an introduction to the philosophy of mathematics but at the same time also uses logical methods heavily.

3. Leibniz' own logical calculus of concepts is an early example of formal philosophy. See Leitgeb (2013) for more on the philosophical-historical background of mathematical philosophy.

4. For more on the explication of concepts, see Chapter 1 of R. Carnap, *Logical Foundations of Probability*, London: Routledge and Kegan Paul, 1950.

5. Of course, also the *philosophy of* physics can benefit from formal methods: see, e.g., Suppes (2002), for the application of set-theoretic methods in the philosophy of physics. See, e.g., the editorial introduction by J. van Benthem and S. Smets, «New Logical Perspectives on Physics», of the special issue on *Logic meets Physics*, *Synthese* 186/3 (2012), for an overview of the logical study of physics.

6. See, e.g., Chapter 12 of T. Williamson (2000).

7. See, e.g., Edgington (1985) for one possible seminal way out of the paradox.

8. See Zalta (1983). Zalta's theory can e.g. be employed to reconstruct Leibniz' logical calculus of concepts that was mentioned at the beginning: see Zalta (2000).

9. For more examples of mathematical philosophy, have a look at my short presentation in the *Mathematics, Philosophy and Mathematical Philosophy* panel of the corresponding workshop at the International Center of Formal Ontology at Wroclaw, Poland, in 2018. See: https://www.youtube.com/watch?v=OltsN2eDDJw. For possible ways of implementing mathematical philosophy in the research and teaching at a university institution,

see, e.g., the Munich Center for Mathematical Philosophy at LMU Munich, https://www.mcmp.philosophie.uni-muenchen.de/index.html, and the Institute of Logic, Language and Computation at the University of Amsterdam, https://www.illc.uva.nl/.

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