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Syntax as a hybrid object in the *minimalist program*: a categorial approach

Resumen: Partiendo de las gramáticas categoriales convencionales (representaciones proposicionales de derivación Lambekiana), Song interpreta categóricamente los átomos sintácticos de la maquinaria generativa. Fusionar (to merge) y categorizar se vuelven lo mismo: las representaciones proposicionales constituyen la última capa de una estructura de abstracción multinivel que genera formas de combinabilidad y types locales.

Sostengo que es posibles brindar interpretaciones duales en cualquier nivel de análisis inferior a aquel de las frases completas: algunas aplicaciones de la CT en filosofía de la ciencia, particularmente el Teorema de dualidad de Stone, muestran que los isomorfismos entre modelos semánticos corresponden a equivalencias categoriales sintácticas.

Finalmente, la arquitectura de los punteros semánticos (semantic pointers) podría proporcionar una base neurobiológica consistente para las abstracciones.

Palabras-clave: Teoría de categorías, Lingüística, Gramáticas generativas, Interfaz sintaxis-semántica, Filosofía científica

Abstract: Departing from the mainstream categorial grammars (full-propositional,

Lambek-like representations), Song interprets categorially the syntactic atoms of the generative machinery. Merging and categorizing become the same: the propositional representations constitute the last layer of a multilevel abstraction-structure generating local types and combinability features.

I argue that dual interpretations are possible at any level of analysis under that of complete clauses: some CT-applications in philosophy of science, particularly the Stone duality theorem, show that the isomorphisms between semantic models correspond to syntactic categorial equivalences.

Finally, the semantic pointers' architecture could provide a consistent neurobiological basis for the abstractions.

Keywords: Category Theory, Linguistics, Generative Grammars, Syntax-Semantics Interface, Scientific Philosophy

1. Actual employment of CT in cognitive sciences: some remarks

We recognize two main areas of application of CT in Cognitive Sciences; both emerged during the second half of the 20th century,



developed increasingly during the last decades, and are considered nowadays well-defined research fields: a) Neural Networks Representations¹ and b) Linguistics' Formal Semantics, specifically Natural Language Processing and Computational Linguistics². Additionally, some general approaches to the whole domain of cognition, proposing theories of principles, could be listed in the survey³.

More recently, some advanced projects aim to a) establish a finer-grained application of CT to natural language, conveying a mathematization of the differences among syntactic categories, and deepening and extending in this way the traditional Lambek-inspired grammars, and/or b) formally specify a link between neural representation and (linguistic) abstraction-modeling.

The present paper is a contribution in both directions, although the endeavor to meet the second challenge is limited to a sort of provisional draft. At the same time, I propose a hopefully acceptable solution to some actual theoretical frictions, which constitute the residuals of the old syntactic vs. semantic primacy contraposition. Although it is not controversial that this contrast weakened in more recent times (particularly after the diffusion of the *Minimalist Program*), a univocal and non-controversial definition of *how* the relation between these two main dimensions has to be intended continues to lack.

The first aspect can be approached by adopting Song's point of view, expressed in his dissertation in 2019: *On the Formal Flexibility of Syntactic Categories*. Some philosophical details will be given in the sixth paragraph of this article, and formal descriptions in the seventh. For the moment it's enough to underscore, that Song, following to some extent the paths of Cohen & Lefebvre (2005), defines *categorization* as the most fundamental cognitive operation, and argues that we could and should formalize it in the precise terms of Category Theory. It is anyway not clear, in his text, where the difference between linguistic and conceptual categorization lies - given that today is no more controversial conceiving *conceptualization without language* as indeed possible and worth modeling⁴.

To better focus on the second aspect and propose some guidelines for future models, I

will comment briefly on some works of Thagard, Block, Do and Hasselmo. Also in the research area constituted by the abstraction's representation through semantic pointers and neural networks, it results quite evident that in recent times the semantic dimension of language-driven conceptualization has been considered more important than ever before⁵. The use of CT and Lambda-Calculus by Do and Hasselmo offers a formal link with the last part of the present work, specifically with the topological *locales* representing the *contents* of the CT-formalized syntactic categories. The pivotal element, for us, is here the famous *Stone's Duality Theorem*, which (a) - maps each propositional theory to the topological space of its models; (b) - shows that the most interesting relations between models correspond to syntactic relations between theories: logical semantics is dual to logical syntax. The last two paragraphs of the present article are dedicated to this complex topic, advancing a kind of - hopefully useful - «working hypothesis».

The third point, leading us directly to the historical ground of the linguistic debates, can be faced first. Following the old syntax-semantics divide, we can track two opposite tendencies - not wholly incompatible, it seems to me if taken with enough flexibility.

1) The book *From Etymology to Pragmatics*, written by Eve Sweetser, constituted a great source of inspiration for the then-new-born cognitive linguistics, and some fundamental perspectives exposed in the text maintain almost the same weight as in 1990, the date of their appearance (the proposal of a systematic analysis of polysemy and metaphorical extensions of meaning, for example). A fundamental tenet of Sweetser's interpretation of semantics is that it should be intended as a direct manifestation of our general cognitive capacities: prelinguistic forms of conceptualization seem to permeate the linguistic practices without essentially changing their original structure:

[O]ur linguistic system is inextricably interwoven with the rest of our physical and cognitive selves. (...) [M]any aspects of language become much simpler when viewed in the collective light of the human

sciences: the study of human culture and cognition is frighteningly broad as field, but there is no point in pretending the autonomy of language if such a pretense obscures real explanatory possibilities. (Sweetser 1990, 6)

At the same time, the linguistic analysis remains formalistic-driven, because general patterns of change, extension, and simplification are factually traceable, permitting the expression of universal characteristics. So it is possible to maintain the generality level, to which linguistics has aimed since ever, and at the same time to overcome some old, only difficultly acceptable relativist tendencies of semanticism.

2) The great linguist and Montagovian scholar Barbara Partee, by contrast, writes that

the closeness of formal semantics to general cognition can certainly be questioned. Formal semantics is itself a highly structured language-specific system whose interface with general cognitive systems may be almost as indirect as their interface with syntax (Partee 2005, 38)

Partee accepts some semanticist principles, criticizing the old definitions of syntactic primacy defended by the early generativists, but her steps beyond the original Chomskian project are directed, overall and in pure Montague's spirit, to render an interpretation of natural language's semantics in logic-mathematical terms, which means recognizing a central role to the principle of compositionality, and with it to the truth-functional semantics⁶. If we follow Partee, we get acquainted with the first general applications of CT to linguistics, given today's well-recognized relation between Montagovian semantics and Lambek's grammars⁷: at the level of full propositional representations, indeed, syntax and semantics are generally recognized by Category Theorists as two faces of the same coin, as Montague argued.

The criticism raised by Sweetser, not explicitly directed to Partee, to the compositional semantics that emerged in the generative tradition during the late '70es, couldn't be clearer:

recent logically based (especially Montagovian) semantic work has largely ignored lexical semantics, preferring to assu-

me that the predicates involved in semantic logical structure could somehow be defined, and to concentrate on the compositional regularities of combining lexical units. (Sweetser 1990, 15)

Is it possible to maintain the extremely useful and philosophically unavoidable principle of compositionality⁸, but at the same time recognize the importance of cognitive aspects of linguistic information and transformational patterns which cease to be visible in a purely truth-functional representation, and give rise to a richer semantic frame?

I propose a model, that I consider apt to overcome this late contrast - taken for granted that the old, rigid contraposition, which presupposed in both cases an explanatorily and epistemologically strong concept of primacy, has progressively lost its original grip. Starting from a basic epistemological conception, it's possible to recognize two different levels of meaning; by defining them, I take advantage of the logical-philosophical concepts of intensionality and hyperintensionality.

But before it's necessary to analyze a little bit more in-depth the «semantic(ist) trouble» which arose along the history of scientific linguistics.

2. Semantics

The best way to summarize the contrast between generativists and semanticists is probably to adopt as a narrative frame the widespread acceptance of the theoretical model of *Deep vs. Surface Structures* during the late '60s. On this basis, we can understand in its full depth the counterposition of Chomsky's *Lexicalist Hypothesis* to the so-called *Katz-Postal Principle*⁹.

The initial accordance between the semantics-specialized scholars and those focussing syntax was pervasive and well-motivated: *Deep Structures* allowed - and precisely because they conveyed a new, central role to the semantic interpretations - a wholly unexplored dimension of analysis, and offered at the same time an easy solution to some emerging theoretical problems. Harris writes:

Syntactic Structures outlined a dramatically new model of linguistics, a rule-based, procedural grammar that built sentences which could be altered or combined by transformations into other sentences. From that foundation, and with help from a growing community of scholars, Chomsky developed a more detailed, technically sophisticated, and richer grammatical model. (Harris 1993(2021), 16-17)

In this climate, the Katz-Postal Principle, «which says that transformations are semantically transparent, that they have no impact on meaning, preserving it from the underlying structure up through to speech» (Harris 1993 (2021), 44), can be seen as an expression of the already common practices of analysis:

The Katz-Post Principle guarantees a free ride from ideas to soundwaves, which makes Deep Structure a kind of universal semantic solvent, dissolving the welter of problems that had long kept meaning at bay in linguistics. (Harris 1993 (2021), 17)

The later adoption of Lexicalism by Chomsky's adepts¹⁰, pointing in the opposite direction, fell like an anathema to those usual practices.

Chomsky's formulation of Lexicalism rapidly came to mean, in Lakoff's attempt to firm it up, that words 'may not change category in the course of a transformational derivation'; or in Jackendoff's version, 'transformations do not perform derivational morphology': once a category, that is, always a category. This move (1) immediately declares a huge swath of the Abstract Syntax program illegitimate and (2) reverses the historical course of Transformational Grammar. (...) All parameters of variation are attributable to differences in the features of particular items (e.g., the functional heads) in the lexicon. (Harris 1993(2021), 116)

Cinque and Rizzi justify the principle on the basis of the idea, that language-conveyed information is necessarily linked to a particular category of representation and particular structuring

rules¹¹. Nonetheless, the divergencies were destined to generate a secession inside of linguistics, counterposing the semantics-oriented linguists and those faithful to Chomsky's position, a fracture that grew till the emergence of two mutually inconsistent research projects.

The semantics-oriented linguists, indeed, proposed in the 1976¹² a completely new perspective, grounding on general cognitive principles and basic forms of conceptualization a kind of «semantics-first hypothesis»; metaphorization-processes became a key-element for the empirical research¹³. This current - hoping to describe the facts in a simple way without oversimplifying them - took the form of the modern CxG and Cognitive Linguistics (with capital letter), dedicating numerous projects to the local typologies, etymologies, and genealogies, and frequently overlapping not only with cognitive psychology, but also with ethnological and anthropological research. From the opposite stream emerged contemporary bio-linguistics, maintaining a stronger accent on the modular nature of a syntax-based *Faculty of Language*. Anyway, it is possible to observe, among generativists, a progressive acceptance of some semantic imports of the adversaries, often without a clear and full admission of their integrations in their own programs. The *Minimalist Program*, indeed, sees a much more complementary conception of syntax and semantics, and the applications of CT in this theoretical frame show systematically the more or less tacit acceptance of the Montagovian syntax-semantics correspondence¹⁴. Harris writes:

Chomsky had attracted a number of people with training in mathematics and philosophical logic to linguistics (including Partee), people who could appreciate the power of Montague's work in ways that previous generations of linguists might not have been able. Montague Semantics grew rather quickly, with notable refugees from Generative Semantics (Lauri Karttunen, David Dowty, and Emmon Bach), along with Computational Linguists, independistes, and various philosophers of language and logic. (Harris, 1993 (2021), 270)

I'm persuaded that the cartographical projects, although maintaining some original syntactician principles, presuppose the same duality (see paragraphs 5-7 of this article).

The authors quoted in the first paragraph, Sweetser and Partee, were doubtless searching for a synthesis between the two currents, although leaning in opposite directions. Respectively, Sweetser appears more semanticist, recognizing the central importance of a broad lexical semantics in context (relation syntax-lexicon), Partee more well-grounded in the logic-philosophical compositional principle, ruling the actual use and linking intrinsically meaning to truth conditions. Both raise important critics and proposals of change to their groups of proceeding, presenting some arguments that we include in our interpretation and our model.

Another pillar for our critical reconstruction of the debate and our possible epistemological solution of the contrast is the work of Wiltschko, in particular her book *The universal structure of categories: Towards a formal typology*.

There are two main reasons for this preference: the author recognizes, with unique precision and a clear consciousness of the heavy theoretical implications, the passage from a rigid structural to a much more functionalist stance in the description of syntactic patterns. This tendency, Wiltschko assures us, is traceable in the whole world of scientific linguistics. She writes: «functionalism denies the existence of universally valid categorial distinctions in favor of a prototype approach. In addition, in the USH [Universal Spine Hypothesis], formal criteria such as c-command are not replaced by functional ones, but instead they are supplemented in this way. In sum, the USH makes available the function as a heuristic to identify the absolute position of a particular UoL (Unity of Language)» (Wiltschko 2014, 97). Secondly, Wiltschko describes and models very clearly the kind of categorization that we need today to pursue linguistic research in all areas, presupposing some non-trivial common grounds. Her definition of *functional categories* was explicitly taken as a theoretical basis by Song, our last and main source of inspiration. In other words: Wiltschko's concept of linguistic (syntactic) categories admits

and encourages their own formalization in the terms and forms of Category Theory.

3. Knowledge and Semantics, Cognition and Non-Semantic Components of Meaning

Given this historical background, I can offer a schematization of *knowledge* (limited knowledge from limited cognitive abilities), which hopefully, per transposition, can throw light on a) *linguistic conceptualization*, b) *narrow semantics* and c) *full (rich) semantics*, differentiating the last two in a sensible way. I proposed the fundamental elements of this epistemological model in my article *From Analytic to Scientific Philosophy*, of 2022. Inspired by Frege's, Williamson's, and Block's work, I introduced a *limited-knowledge-relation*, which I labeled Δ , operating on «the true proposition p. We can then define a (...) generalizable cognitive access $S\Delta p$, where S is a mental state: $S\Delta p$ is the conceptualization (...) of p that a subject can form if she, being in S, knows p through the general cognitive patterns, i.e. inside of the cognitive limits, Δ » (Boccafogli 2022, 145). It is important to notice that the proposition p is not intended here in a *Russellian*, i.e. structured, sense. On the contrary, it's intended as anything that could characterize a *propositional attitude* independently from the way in which we define the latter (many possible ways do indeed exist, from simply dispositional to complex representational accounts). In this specific case, p it's the object of knowledge - considered, as Williamson does, as a primitive mental state. If p is known, it's true; and if it's true, *it's (instantiated by) a fact*: correspondingly, we leave open also the definition of «fact», which could be unstructured and/or partially representable through deeply different structures. In this sense, a prelinguistic knowledge is indeed possible: a little child *knows* that her mother loves her, a dog *knows* that his owner arrives, etc.

We can pass to structured propositions using the same basic schema $S\Delta p$, if we want to accede

to a properly linguistic representation level. Let's sketch narrow semantics in this way: p is the proper content (the structured proposition), while Δ makes explicit how a particular expression « p » allows the access to p being the cognitive subject in S . In other words, the structure of p must be incorporated in Δ through a particular « p ».

Refined version: « $S(Cp)\Delta$ is the conceptualization of p that a subject can form if she, being in S , knows p under the possibilities and the constraints expressed in $(Cp)\Delta$, the Categorical structure of ' p ' embedded in Δ ». Where:

1. Δ is the knowledge-relation expressing all cognitive patterns/limits involved, including the specific expression of p (or its corresponding subclass, we will see), which can be labelled «singular expression» and can be interpreted as a determinate instantiation of Cp : the operator is hyperintensional.

2. With «Categorical (*syntactic-semantic*) structure Cp » we mean the application to p of a C-theory of grammar, i.e. a complex structure common to the whole intensional equivalence class of possible sentences $\{|p|\}$: the *structured proposition*. Consequently, it doesn't formally coincide with the clause « p », which could instead, like the other equivalent expressions, *trigger* the mental processes *corresponding* to Cp (understanding) or *derive* from them (production). The postulation of a correspondence between mental processes and categorical structures does not mean that we intend the latter as mental and/or possibly neural in nature. *Categorical structures are intended as abstract, leaving open the problem of their material instantiation.*

3. The cognitive subject knows p through « p -categorical» bounds and limits. Choosing a Tarskian scheme, we want to underscore that we can't ground further this Aristotelian mirroring of facts and contents. Here we will not treat directly the metaphysical implications of our semantic conception, limiting ourselves to avoid any kind of idealism and/or antirealism: linguistically expressed facts are for us just a subset of worldly facts.

A central tenet is that in a successive moment / possible situation $S|\Delta|p|$, independent from the involved cognitive subject(s), Δ itself can

be treated at its turn as a proposition $\Delta 0 := p1$, i.e. as a content which can be pointed to, and consequently materially introduced, in different ways (through different, equivalent expressions): $Sx\Delta x\Delta 0$. We can limitedly grasp and come to know forms and details of our limited capacity of knowledge: limitedness and hyperintensionality go on a pair. In this way, besides maintaining the compositional principle, through this simple formalism we can represent a cognitive aspect which together with categorization is considered the most important and distinctive of linguistic conceptualization: that of recursion.

In our model syntax and semantics are given together: categorical structures permit to analyze language employment, i.e. *performances*, before and more than pure linguistic *knowledge* - so they include semantics, excluding primacy-theories. Described through this model, the *rich semantics* presented by Sweetser as the main object of cognitive linguistics, is due to some (partial) inclusion of the *hyperintensional operator* Δ (or analogous ones) in the *semantics* of the expressions: in her interpretation, as we will see more in detail, two expressions which convey (classically) equivalent logical contents, could indeed diverge semantically if placed in different cognitive contexts. We have decided, on the contrary, to draw a fundamental distinction between the *narrow semantic* of an *expression-in-context* and the eventual treatment of the whole complex context in semantic terms (object of another, higher-degree operator).

Anyway, rendering justice to the excellent intuitions at the basis of Sweetser's model, we consider evident that general patterns of derivation among different historical modes of presentation are traceable in an objective way, and that we can investigate them only if we dare a step outside of intensional models in representing language. It's a kind of bipolar tension: on the one side we have the specific (varying) contexts covered by Δ , on the other the fact that conceptual-linguistic Δ always entails, although is never equated with, general categories, permitting both compositionality and intensional equivalence. In this tension we individuate the seed of regular change; to model it, we resort to the help of both historical linguistics and

neurobiology. Every linguistic expression, indeed, can be represented through a structure of patterns (the structured proposition) conjugated to a particular kind of semantic pointers' architectural configuration (see next paragraph); but as a *singular* expression it results also embedded in a cognitively broader context, so that the Cp-dependent semantic pointers-configurations are always contextually associated and also contrasted with other ones, of the same and of different nature. In this way, some kinds of cognitive contexts come to characterize, for frequent associations of some neural patterns with the singular expressions (or with particular classes of them), common linguistic uses and emergent mental habits, becoming in this way *pragmatic components of meaning*. This «indirect value» can be presented as a scientific topic grounding the aspects of broadness/richness which semanticists supposed to extend the truth-functional semantic basis.

Our model entails what follows. 1) Under the class of intensionally and structurally equivalent expressions, we have possibly subclasses of linguistic expressions playing the same hyperintensional role, i.e. permitting another level of more «internal» and restricted cognitive equalities, beyond that of logical equivalence. 2) Departing from pure intensions for a finer-grained interpretation, we should represent the different levels of granularity of the structured propositions through different categories (for example: phrase-internal and full-propositional). Structured intensionality should anyway never collapse on hyperintensionality. 3) A neurological counterpart is included in the model. Relatively complex configurations of semantic pointers (both pre-linguistic and language-related concepts) derive both ontogenetically (infant's acquisition) and phylogenetically (historical growth of conceptual complexity in a community) from other simpler ones through a kind of mixing-process, which operates a selection and a composition of features of the «older» components. More in details, rich and broad experiential/perceptual contexts (informational overflow) are systematically simplified and idealized at the cognitive level, so to permit the reference to relatively few, simple, and persistent types of objects/facts/properties,

which serve systematically as a basis for upper configurations. In this way we can depict the internal counterpart of semantic compositionality and linguistic categorization (correspondence between pointers and equivalence classes of intensional structured expressions). This activity, at least sometimes, results partially conscious¹⁵. At the same time, as seen, in every cognitive context we have an association and possibly a contraposition of the Cp-related pointers' configuration with other pointers, which partially depend on the singular expressions (contextual instantiations).

We will try to analyze these results a little more in-depth in our working hypothesis. But to do it, it is necessary to schematize in advance and with more detail our concept of semantics, given that it's situated at the crossing point of philosophy of language, linguistics, and logic¹⁶.

As Frege explains in his most famous article (*Über Sinn und Bedeutung*, 1892), coreferential expressions can have different *senses*. We can take as examples *i*: «Aristotle's master» and *ii*: «the author of Republic». The difference between extensional and intensional contexts can be made explicit through Leibniz's Principle of interchangeability *salva veritate*: in «x founded the Academia» both expressions can complete the clause in substitution of x without originating divergences in truth-values. Different would be the case of clauses introduced by propositional attitudes, such as «Mr. Y knows/believes/remembers/etc. that x founded the Academia». There is no logical or metaphysical impossibility in the credence that x can fit the description *i* without matching at the same time *ii*: the intensional contexts can be indeed represented as *possible* worlds where these expressions have different extensions, i.e. give rise to true propositions if predicated of sets of individuals not matching the actual one.

Intensionality (and with it Fregean *sense*) has to be considered as an objective, not internal-representational instance, like in the telescope's example¹⁷. It does not coincide, as a consequence, with the *Internal Language IL* postulated by Chomsky. *Epistemic and/or doxastic logics*, treating respectively contents of the speaker's belief and knowledge, are constructed following

the same objective intensional principles. It follows, that from the chosen point of view states of belief/knowledge can be objectively depicted (leaving momentarily aside the problem of a *logical omniscience* of the speaker).

Taking into account what has been called «the intensional revolution» in philosophy¹⁸, we finally use the structure of categories to describe and formalize mental processes of linguistic comprehension and production, but more fundamentally we employ it to represent, in terms of types of information, the relations between a sentence and its truth-values in possible worlds. The problem is that we consider the correspondent proposition (the function from worlds to truth values) as a structured entity: a categorial interpretation can eventually help to find a compromise between structured propositions and truth-value semantics.

Implicatures which are not describable as strictly subjective (such as typical emotional tracts, for example, or historically framed associations of ideas, etc.) result impossible, or at least very problematic, to include in the *rich-semantic-frame* (to quote a sentence *p* in a new linguistic frame *f*, for example, does not necessarily convey the original emotional associations and reaction proper of *p*, although we would not say, I surmise, that the meaning of *p* has changed in *f*). The difficulties become even broader if we take into account the frequency of hyperintensional contexts in which the linguist/logician/philosopher, if she adopts this approach, would attribute illogicality to the speaker¹⁹. The two expressions «we have the 60% of possibility of winning» and «we have the 40% of possibility of losing (= not winning) », for example, although logically equivalent, i.e. necessarily compositionally identical in terms of truth-functional semantics, are distinct elements of the same *class of intensions*, i.e. they are, after all, cognitively and materially different when considered as singular expressions. In other words, although elements of the same class of intensions, they are not elements of the same hyperintensional subclass. The point is that this difference should be considered as «slight» from the point of view of meaning: it shouldn't reach the level of semantic

content, being it structured or not, as a *rich-semantic-frame* would imply.

Our definition of «cognitive differences» needs now two specifications: a) postulating them doesn't mean, that the cognitive subject reasons illogically, but just that it is possible that she fails to recognize the referential equivalence of the expressions. The extensional situation is perfectly describable - given that the metaphoric which proved to be the best one to represent concepts is indeed the spatial one²⁰ - in terms of different paths leading to the same place: arriving from different locations, one can fail to recognize two or more different modes of presentation of it as presentations of the *same place*, especially at a first glance (without reflecting on it). We conceive this difference as semantic²¹. On the contrary - and differently from a purely intensional equivalence, which means just having the same truth value in all possible worlds - classes of *intensionally and structurally* equivalent *expressions* would be describable as slightly different presentations of the same path, arriving at the same point of the «semantic map», i.e. offering *the same* perspective on the arrival-place (substantive reference). We consider the «slight differences» among them as purely pragmatic.

Developing the metaphor, this distinction could be presented in terms of logical possibilities and necessities, representing the first image the *possibility* of different worlds being different the paths, the second the *necessity* of the world being the same, being the path the same. This distinction, although essentially metaphysic, must be hardwired in our comprehension of grammar, meaning and logic.

b) In the next paragraphs we will try to intuitively sketch the here introduced *classes of structured and intensionally* equivalent *expressions* in terms of *flexible categories*. Some hyperintensional differences²² are recognizable in this variety. Anyway, following Lewis' maxim (no truth value = no semantics²³), these internal differences should not be confused with the different levels of abstraction of structured propositions, which are on the contrary a future object of categorial interpretation. This difference in granularity can be well represented by

the two necessary correspondent values in the percentage's example, where the semantics of the composing parts (or phrases) are evidently different structured expressions, but the whole propositions, conceived as functions of the phrases, should be considered equivalent. It follows, that *intensionally equivalent structured expressions* (here the main difference from the logicians' conception) *are not necessarily (or not primarily) logically equivalent; they are categorically equivalent* instead. In other words: they are logically equivalent in the sense of *categorical logic*.

In this way, at a purely theoretical level, it seems to me that we are able to compose the contrast presented in the previous paragraph. Trying to overcome the opposition between the two old primacy principles in a kind of synthesis, we can argue that the functional character of both the semantic pointers and the flexible-categories structure allows for dual models (syntactic-semantic) for the neurobiological and the linguistic-theoretical representations. Now it's time to show how pointers and categories presuppose a selection of different kinds of information.

So interpreted, the linguistic-theoretical and the neurobiological domains present strong analogies, and Thagard, Block, Hasselmo & Do offer us wonderful models to bridge the two.

4. Semantic Pointers

... So let's cross the threshold of neurosciences - although just to gain an approximative map of a restricted area. Indeed, I consider this paragraph as a kind of draft, made up of a patchwork of essential quotes, for future interdisciplinary works. It's undoubtedly useful, and necessary for the topics and arguments presented here; but at the same time, it requires a profound extension and detailed description, given the extreme complexity of the argument.

Talking about semantic pointers, we suggested the image of the *material* constitution of representational activities (and successively linguistic reference) as a kind of pointing, relating certain complex *stimuli* to acquired categorization patterns. Ascending to more abstract

complex contents, we have mentioned «a kind of mixing process, which operates a selection and a composition of features of the simple component». We have also added, that «[t]his activity, at least sometimes, results partially conscious».

Block, for example, analyses the context of working identification of moving perceptual objects («object-files»); he argues that

The object files of working memory and singular thought enclose the perceptual materials from perceptual object files in a cognitive envelope and in addition transform the perceptual information, often misrepresenting some aspects of the stimulus in order to make other aspects of the stimulus easier to use for a specific task. That is the problem for grounding singular thought. (Block 2022, 1)

The contextually relevant cognitive aspects of *some object/event* are selected, simplified, and *composed* to permit a stable access to the same entity under specific conditions. Our storage- and computational capacities are selective insofar they are limited, and we can reach always new object-files' representations through different compositions of previous simpler ones.

In this way, we are allowed to eliminate the idea of an always present «general space of representation» processing all information in favor of the selectivity and the competition structure of the semantic pointers (Eliasmith 1999). Thagard writes in this spirit:

The Semantic Pointer Architecture does not require any central processor or blackboard for computation, which is accomplished by interacting neurons organized into groups. Some of these groups, or groups of groups, provide hubs for communication among different parts of the brain that are variously called «neural hubs», «convergence zones» or «association areas». Hence, the brain is very different from modern computers that have at most a small number of processors through which information flows.

Another major way in which brains differ from computers is that memory does not

work by storing exact digital encodings but rather by transforming sensory experiences into neural connections that are only approximations to the experiences. (Thagard 2019, 37)

In comparison to computers, a slower performance rate and a lower storage capacity respond to a higher complexity of our neural productions. The complementarity between Block's and Thagard's approaches becomes evident in these lines:

A semantic pointer is a special kind of neural representation – pattern of firing in a population of neurons – that is capable of operating both as a symbol and as a compressed version of sensory and motor representations. For example, people's neural concept of chocolate unpacks (decompresses) into sensory representations of sweetness, texture, and so on, while allowing the semantic pointer to figure in inferences such as that you should not eat much chocolate because it is a kind of candy. Semantic pointers are formed by binding together simpler representations, where binding is a neural process that compresses information into a more compact form suitable for manipulation. (Thagard and Steward 2013, 75)

This kind of selective simplification could be considered the proper basis of concept-formation in general, beyond any specific task in singular thoughts or in working memory's contexts. *Postulating* a stable objective content permits indeed to link internal representation with semantic compositionality. The non-representational aspects involved, i.e. the initial rich perceptions, are processed so to isolate some relevant features, and precisely in this way they permit the access to linguistic structures: criteria of identity and classification (or categorization) are given together. The simplification (or reduction) is indeed what permits recognition, re-presentation and combination of the *same objects intended as particular tokens of general types - the basis for symbolic representations and consequently for objective and structured knowledge*. Finally, we consider flexible linguistic categories to be associated with constant

pointers' configurations. The different singular expressions (instantiations), instead, are possibly associated with different pointers / competitors. So Hasselmo & Do:

people learn a concept when they do pattern recognition. They can parse a character into its most important parts and relations (compositionality), apply them to different situations (systematicity), and generate new examples (productivity). Deep neural network models have yet to demonstrate these abilities (Marcus 2018, 67-71)

Biological neural network models on the other hand have been used to model various forms of attractor dynamics (...), supported by evidence from a growing number of large-scale neurophysiological recording and manipulation studies (...). Essentially, these networks are dynamical systems that over time settle to a stable pattern termed 'attractor'. That pattern might be stationary, cyclic, or chaotic. The networks' state at stability could then be described as residing on some low-dimensional manifold (point, line, circle, plane, toroid, etc.), which enables various robust and reliable information processing capabilities like noise reduction (...), categorization (...), integration(...), or memorization (...). Interestingly, these observations are consistent with the Manifold Hypothesis (...) in machine learning, which states that the embeddings of high-dimensional real-world data tend to lie in the vicinity of a low dimensional manifold. The challenge for the connectionists then is to establish the neural operations that can manipulate manifolds by controlling and constructing attractors, perhaps by introducing translation to move the network's state to another location inside or outside of an existing attractor, or by transforming or changing the kind of attractor the network is implementing on-the-fly (...) (576-593)

In the last paragraph, in very general terms, we will see how, in the case of linguistically structured abstraction processes, the neurobiological construction of attractors corresponds to the creative manipulation of

categorial (syntactic-semantic) patterns: Do and Hasselmo adopt indeed a category-theoretical representation, so that it results possible, in principle, to trace an isomorphism between the linguistic and the neurobiological descriptions. Their approach too can be considered largely functionalist, including arguments on semantics, social determination of meaning, and pragmatics. The pointers and the attractors' theories can fit very well with each other, in particular in the representation of the sameness and the equivalence of the abstractions, due to our constant recognition and use of similarity and identity patterns (building up new pointers/attractors on the base of old ones), and in the representation of the competition among groups.

In this frame, the recognition of an extensional equivalence between expressions *a parte subiecti* can be easily interpreted in terms of a net of frequent associations (inductive reasoning) and logical inferences (deductive reasoning), while the intensional structured equivalence presupposes equivalence classes for both attractors and categories.

5. Wiltschko: The Categories

I consider Wiltschko's book a cornerstone for contemporary linguistics, in particular because it provides a very good argument in favor of a functionalist interpretation of the epistemological principles, on which Chomsky's *Minimalist Program* seems to be grounded. Instead of a Universal Grammar founded on possible universal syntactic patterns (maybe deep-structural) or universal ground experiences giving rise to metaphorical extensions, the author focuses on the main functions that a linguistic expression has to absolve, framing in this way a methodological normativity constraining the syntactic and semantic modelings.

Tackling the problem of universality, she writes that what is usually defined as a grammatical category is not a homogeneous class. We can distinguish words, morphemes, features, and types of construction. So long these are categorizable, they can be called Units of Language (UoL).

Categorizable Units of Language:

- a) Words: determiners, complementizers, auxiliaries, ...
- b) Morphemes: possessive, progressive, ...
- c) Features: tense, number, case, ...
- d) Clause-types: imperative, subjunctive, ...

We talk about a category when we can make generalizations over the distribution of a whole set of UoLs. For example, if we know that a word belongs to a certain category *c*, then we automatically know the distribution of this word. Crucially, this distribution cannot be determined based on either the meaning or the sound of the word. But where does this categorial identity come from? Is it part of a universal repository of categories that is part of our genetic endowment, i.e., part of a universal grammar? Or does it emerge as a matter of language use? (Wiltschko 2014, 1).

The idea of attributing a grounding role to the categorization was already present in the generative traditions from the '60s on, also in some of Chomsky's writings about the Universal Basis Hypothesis²⁴. Wiltschko's stance can be described in terms of the passage from a Universal Basis Hypothesis to a Universal Spine Hypothesis, being the latter introduced redefining «categorization» in relation to the UoLs:

(...) there are certain formal characteristics of grammatical categories that cut across language-specific patterns in that they go beyond individual sound-meaning associations. These characteristics concern the way UoLs relate to their interpretation. What we observe is that the categorial identity *c* of a given UoL (i.e., its distribution) plays a critical role in the way this UoL relates to its interpretation. That is, the relation between a UoL and its interpretation is mediated by its categorial identity *c*. This suggests that the existence of *c* is a linguistic reality (Wiltschko 2014, 3).

Aspects of cartography are in this sense associated with typological on-the-field research and as a consequence included in Wiltschko's interpretation and project:

Universality of linearization and categorization is explicitly reintroduced in Cinque (1999), a framework that has come to be known as cartography and which has become a prolific research agenda. Its main thesis is that all languages have the same functional structure and that every projection in this structure is associated with a precise semantic interpretation (Wiltschko 2014, 11).

But we need some more steps to resolve the problem of universality. UoLs are indeed parts of a well-defined grammar, which at its turn appears to be indissolubly linked to a particular language/class of languages/root language (as Benveniste wonderfully showed). Previous attempts at bottom-up approaches to the problem of universality took usually material (substantive) comparisons as a basis, considering the general morphological and typological differences presented by historically (spatiotemporally) distant grammars too hard to bridge. The conception of the locality of grammars allowed authors such as Haspelmath to argue that universal patterns had to be found at the level of the material elements (individually separable semantic traits), not that of local categories (in Wiltschko's terminology: UoLs): a «comparison cannot be category-based, but must be substance-based because substance (unlike categories) is universal.» (Haspelmath 2007, 119). But which kind of categories does he have in mind?

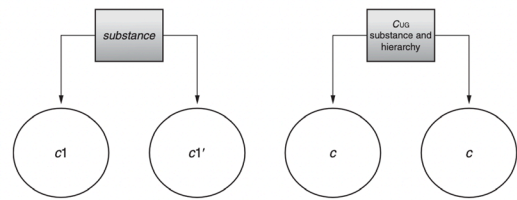


Figure 1.8 Substance-based comparisons

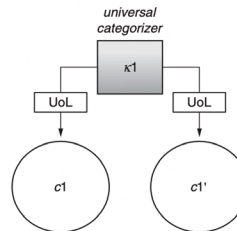


Figure 1.9 Comparison based on κ

(Wiltschko 2014, 35).

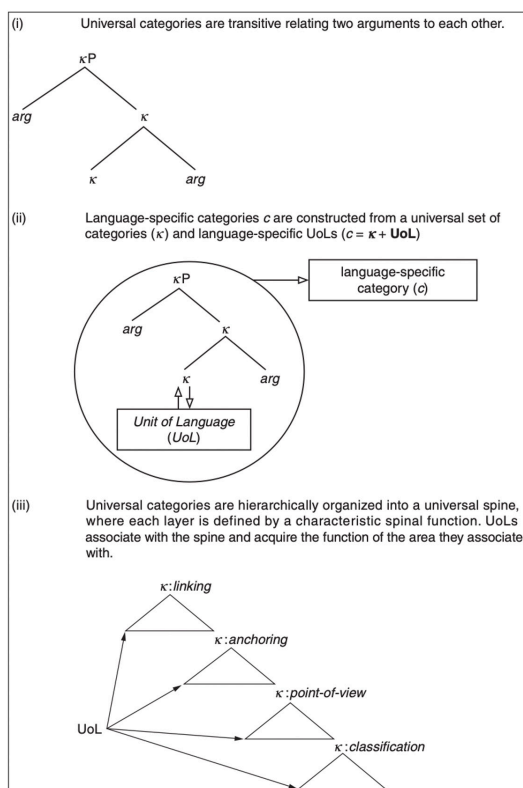
Postulating the existence of a *Unities-of-Language-categorizer* k and a philosophical grammar intended as a prototypical *tertium comparationis*, Wiltschko goes up from a «substance-based comparison» to a *formal mapping* between more general types triggering abstraction processes. This mapping, as Song (2020) notes, shows exactly the characteristics of categories-formation in Category Theory, establishing which structural aspects should be maintained through the morphisms. We can interpret this model in terms of a general formal ontology, constraining all possible information-processings on the ground of the general form of structured propositions themselves. Wiltschko's proposal, indeed, takes the form of a typological project, which is *formal* essentially «because it is based on the structure of categories, rather than their meaning» (Wiltschko 2014, 34).

We can describe some universal layers (formal categories) based on the cartographic role that they play in typologizing kinds of information. For sure, an attentive reader can already recognize in these groupings the kind of categories I have already postulated at the end of the third paragraph. The compatibility with the pointers-attractors architecture results equally quite evident - in particular in the hierarchical

structure and the analogical treatment of wide classes of constructs.

I assume that the spine is inherently category-neutral. Its verbal or nominal character is derived. The lowest layer (κ :classification) is responsible for the classification of events or individuals; the second layer (κ :point-of-view) is responsible for introducing a viewpoint relative to which the event or individual is presented. The next layer (κ :anchoring) is responsible for anchoring the event or individual to the utterance; this may result in either deictic or anaphoric anchoring. And finally the spinal function of the outermost layer (κ :discourse linking) is to establish a relation between the proposition or referent and the ongoing discourse.

(49) CUG = κ :discourse linking > κ :anchoring > κ :point-of-view > κ :classification (Wiltschko 2014, 28).



(Wiltschko 2014, 28).

6. Song: The Category-Theoretical interpretation of Wiltschko's Categories

Nowadays we want to apply CT to Linguistics at finer granularity levels, under the propositional representation. We want to reach what Chomsky called the «syntactic categories», linking them so close as possible to the «lexical categories». Precisely Wiltschko's model, based as it is on a categorial and (potentially) universal interpretation of the Unities of Language, opens the way for a deeper mathematical interpretation of Generative Grammars in general, because all that we needed for our enterprise was indeed an interpretation permitting to approach syntax and semantics as the two faces of the same coin and to do this from a not-local (not-specific-languages-relative) perspective. The young theoretical linguist and applied mathematician (I would add: «scientific philosopher») Chenchen Song opened this new way in 2019/2020, with his dissertation *On the Formal Flexibility of Syntactic Categories*.

Wiltschko's categories, composing explicitly a *hierarchy*, constitute a *Partially Ordered Set* («Poset»). If we build up arrows connecting them to the substantive expressions instantiating them in a given language, we obtain an *Epi-adjunction*, a case of *Epi-morphism*.

Definition 6.3.3.5 (Galois connection). Given two poset Categories $\mathcal{P} \leq$ and $\mathcal{Q} \sqsubseteq$ and two monotone functions qua Functors $f : \mathcal{P} \leq \rightarrow \mathcal{Q} \sqsubseteq$ and $g : \mathcal{Q} \sqsubseteq \rightarrow \mathcal{P} \leq$. We have an Adjunction $f \dashv g$, commonly called a Galois connection, between $\mathcal{P} \leq$ and $\mathcal{Q} \sqsubseteq$ if $f(p) \sqsubseteq q \Leftrightarrow p \leq g(q)$ (6.3.3.6) (...)

Definition 6.3.3.6 (epi-Adjunction). Given a Galois connection $f \dashv g$ between poset Categories $\mathcal{P} \leq, \mathcal{Q} \sqsubseteq$:

$$\mathcal{P} \leq \xrightleftharpoons[f]{f} \mathcal{Q} \sqsubseteq$$

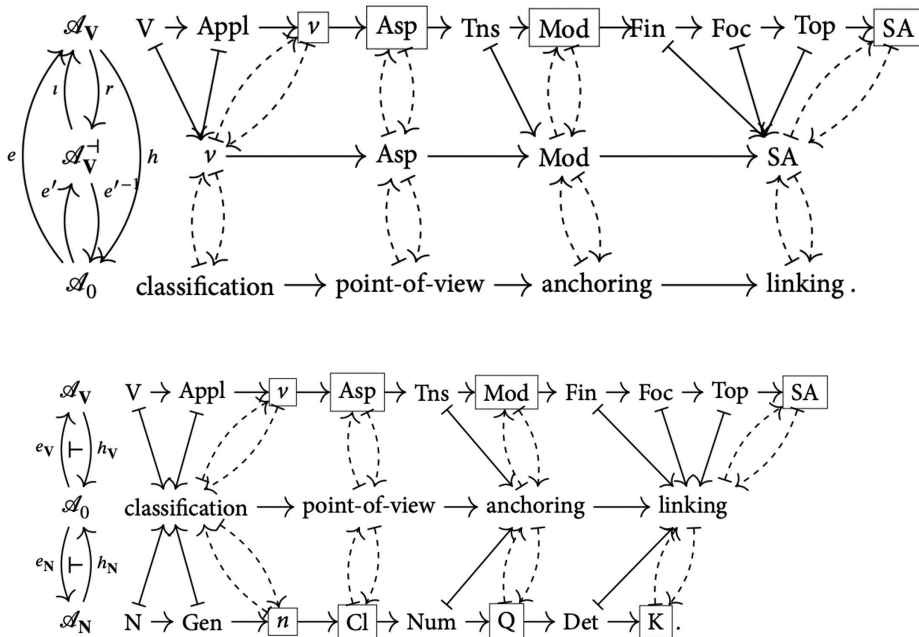
If $\mathcal{Q} \sqsubseteq$ is isomorphic to a subposet of $\mathcal{P} \leq$, then the left adjoint f is surjective, and the situation is called a right perfect Galois connection or

epi-Adjunction. The composite Functor $g \circ f$ is called a closure operation, for it maps $\mathcal{P} \leq$ to its fixpoint set; that is, applying $g \circ f$ twice yields the same result as applying it once. All these notions can be dualized, to a left perfect Galois connection or mono-Adjunction, a kernel or interior operation, and another fixpoint set (Song 2019, 208-209).

Representing graphically the situation, we can group the UoL-Poset around the principal cartographic topics (fix point set) obtaining a reduction of informational distribution (epi- or onto function), and impose a one-to-one function

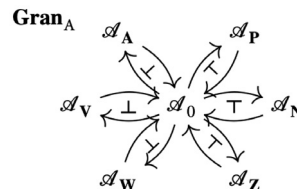
between these last elements and the *spine* (first picture).

At the UoLs level, we obtain in this way a mono-adjunction, which can be described in CT in a dual way, formalizing in a unique structure also the described functor which maps in the opposite direction all the elements of the cartography to the elements of the principal topics (the epi-morphism). We can also include in the example two kinds of phrases, verbal and nominal, conveying the same categories of information (second picture). The functors can be composed through products. On the left side of the picture, we can recognize the functors and their relations:



(Song 2019; 216, 219)

All kinds of phrases can be analogously related to the central Poset, i.e. forming groups of syntactic elements (which obviously convey lexical and semantic information) related to different elements of the *spine*. We can depict this situation about a chosen level of granularity A , with what Song calls «The Flower».



(Song 2019, 220)

The «flexibility» of these so intended syntactic categories, i.e. their multiple possibility of implementation relatively to a specific kind of phrase, permits the definition of an equivalence-class of expressions (not necessarily actual, but often easily traceable in many languages). These equivalent classes vary depending on the chosen granularity level: a future task will be to define in a satisfying way some inter-level relations between categories. This is *exactly* what we anticipated in the third paragraph when we said that «*Intensionally equivalent expressions (...)* are logically equivalent just in the sense of *categorical logic*».

7. Models and Theories: Posets and the Stone-Duality-Theorem

Song's categorial formalization permits us to dare a further step ahead, introducing some categorial conceptions of theory and theoretical content which became widely known in the world of philosophy of science (of physics in particular) thanks to Halvorson and Tsementzis, 2017.

Through their work, Category Theory offered to philosophy of science a general conceptual-mathematical framework. The *syntactic* and the *semantic* interpretations, considered opposite during the whole 20th century, proved indeed complementary in the 2-category of scientific theories, given that 1) the *Th-category* of propositional theories and the *Bool-category* of Boolean algebras are structurally identical; and 2) through the *Stone duality theorem*, *Bool* is proved to be dual to the *Stone-topological subcategory*, so that each propositional (syntactic) theory is mapped to the topological space of its models (possible worlds-sets). In other words, the theorem shows that the isomorphisms between semantic models correspond to syntactic categorial equivalences and that the most interesting topological analogies correspond indeed to syntactic ones.

Finally, the authors of the paper *The Categories of Scientific Theories*, can say, «On the one hand, a semantic presentation of a theory is nonetheless a presentation – written in a mathematical

language. On the other hand, the syntactic category of a theory is a hybrid object, neither purely syntactic nor purely semantic». (Halvorson & Tsementzis 2017, 31): the semantic view is *not* language-independent, and the syntactic one is not free from non-observational world structures, i.e. its laws are not to be interpreted as *a priori* determined.

We argue (following an ideal line of development Lambek-Montague-Partee-Coecke) that a similar syntactic/semantic interpretation is possible also in linguistics. Instead of linking opposite interpretations of theories, we should link opposite interpretations of languages. The disanalogy between languages and theories (sentences and theories are true/false, not languages) is not discriminant in our cognitive frame (knowledge-relations), where structural aspects (models, equivalence, and extendibility) become central. The aprioristic assumption mentioned above talking about the nature of syntax itself (which was considered for a long time the proper vehicle of scientific theories), corresponds if transposed to linguistic milieu, to the *a priori* character that Chomsky attributed to the syntactic elements of the Universal Grammar. *All* contemporary streams in bio-linguistics (and more obviously in cognitive linguistics, proportionally to the degree of compromise they maintain with the original semanticist conception) do not entail a syntactic primacy in the original sense. This does not mean that universal elements, i.e. innate structures independent from experience, are impossible or excluded in principle, but just that they are not to be recognized in substantial syntactic structures. An interpretation of the syntactic categories in Wiltschko-Song sense permits instead to trace some *universal patterns of communication* which result in a well-founded adaptive function.

With these methodological premises in mind, we can proceed as follows.

1. We can, first of all, specify a little bit more in depth the kind of relation that subsists between syntax and semantics under a categorial viewpoint. Olivia Caramello offers probably the best explanation of this topic in her famous article A

topos-theoretic approach to Stone-type dualities, published in 2011.

2. Starting from this point of view, we can transpose to the main topics of comparative linguistics the categorial (syntactic + semantic) Morita equivalence, the best way to compare scientific theories. Morita is a kind of equivalence that is much more flexible than the purely syntactic one and has to do with isomorphisms between the conservative extensions of the theories - in our case of the historical natural languages themselves (historical linguistics, indeed, surveying the history of languages, gives many examples of extensions, which would fit well in our representation).

1 - As Caramello mentions already in her introduction, by the term «Stone-type dualities» «we refer, following the standard terminology, to a class of dualities or equivalences between categories of preordered structures and categories of posets, locales or topological spaces, a class which notably includes the classical Stone duality for Boolean algebras (or, more generally, for distributive lattices), the duality between spatial frames and sober spaces, the equivalence between preorders and Alexandrov spaces (...)» (Caramello 2011, 4).

The categories of preordered structures (the most important necessary relations for the institution of Posets, indeed - specifically *Song's Posets*, in our interests) can be interpreted as corresponding to the syntactic coherent categories (we remember: coherent logic is to be intended as a fragment of classical logic, lacking universal quantification) in which theories can be written. «We have seen that models of small ordered algebraic theories presented by generators and relations *can always be constructed as preordered syntactic categories of generalized propositional theories*» (Caramello 2011, 131 - my italics). Geometric coherent syntactic categories are indeed what permits us to *interpret semantically the underlying «contents» in topological terms, as models*. Halvorson & Tsementzis 2015 and Halvorson 2019 prove that these kinds of geometric coherent categories are indeed the mathematical objects that can best

represent scientific theories - in other words, more in our perspective: they differentiate *grammatically coherent languages against a-grammatical isolated expressions*. Caramello - and this is the central point, deeply correlated, in her own writing, to the *Morita equivalence* - argues:

We can use propositional geometric theories to describe subsets of a ring with particular properties, such as the class of ideals of the ring; the subterminal topology then provides a way of endowing the collection of models of such a theory with a topology such that *the topos of sheaves on the resulting topological space is equivalent to the classifying topos of the propositional theory; also, (...) in many cases of interest, an explicit semantic description of such classifying topos as a topos of sheaves on a poset structure presented by generators and relations with respect to some Grothendieck topology on it.* (Caramello 2011, 142 - my italics)

2 - Let's imagine a language in which someone tells *something that she knows* (the linguistically mediated content p); as seen, the form Δ in which she expresses that content comprehend a *class of intensionally equivalent structured expressions*. We can now compare two languages saying «the same» truth using equivalent kinds (types) of cartographical bits of information: *they are simply specifications of Δ^{25}* .

Let's now think about the concrete possibility that a language is too poor to express what the other says (this condition usually depends on differences in existing lexical contents and vocabulary richness - elements destined to grow in time; but it is not always the case: a language could indeed lack the possibility of expressing a *tense* that the other possesses, i.e. a cartographical type). We can easily and realistically presuppose, that the lacking language is *extensible*, i.e. that it is *possible in principle* to implement some forms, so to express the same content through an extension that does not change its previous grammatical rules. The Spine-Cartography map of the basic expressions, in this image, orients the extensions of the lacking sequence, functioning

precisely like the attractors in Do and Haselmo’s neurobiological model.

The *Morita equivalence* between the two languages can be introduced through the appropriate concept of extension as follows:

Definition 5.2.2 Let $\Sigma \subset \Sigma^+$ be signatures and T a Σ -theory. A Morita extension of T to the signature Σ^+ is a Σ^+ -theory

$$T^+ = T \cup \{\delta s : s \in \Sigma^+ \setminus \Sigma\}$$

that satisfies the following conditions. First, for each symbol $s \in \Sigma^+ \setminus \Sigma$, the sentence δs is an explicit definition of s in terms of Σ . Second, if $\sigma \in \Sigma^+ \setminus \Sigma$ is a sort symbol and $f \in \Sigma^+ \setminus \Sigma$ is a function symbol that is used in the sort definition of σ , then $\delta f = \delta \sigma$. (For example, if σ is defined as a product sort with projections π_1 and π_2 , then $\delta \sigma = \delta \pi_1 = \delta \pi_2$.) And third, if αs is an admissibility condition for a definition δs , then $T \vdash \alpha s$.

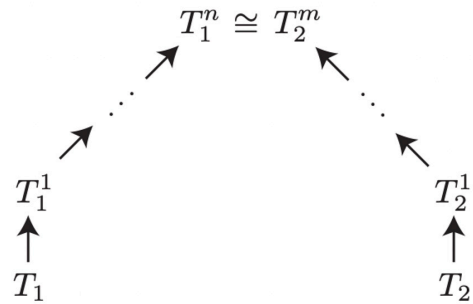
Note that unlike a definitional extension of a theory, a Morita extension can have more sort symbols than the original theory (Halvorson 2019, 135).

The *sorts* about which Halvorson speaks are indeed interpretable as cartographical types. Their function is perfectly identical, here, substituting «theory» with «language». The difference lays in the fact that out types bring about, by definition, purely intensional, structured mapping (definitions of σ as a class of equivalent expressions). Under the same condition, we read this conclusion:

Definition 5.2.4 Let T_1 be a Σ_1 -theory and T_2 a Σ_2 -theory. T_1 and T_2 are Morita equivalent if there are theories $T_{1,1}, \dots, T_{n,1}$ and $T_{1,2}, \dots, T_{m,2}$ that satisfy the following three conditions:

- 1) Each theory $T_{i+1,1}$ is a Morita extension of $T_{i,1}$.
- 2) Each theory $T_{i+1,2}$ is a Morita extension of $T_{i,2}$.
- 3) $T_{n,1}$ and $T_{m,2}$ are logically equivalent Σ -theories with $\Sigma_1 \cup \Sigma_2 \subseteq \Sigma$.

Two theories are Morita equivalent if they have a «common Morita extension». The situation can be pictured as follows, where each arrow in the figure indicates a Morita extension.



(Halvorson 2019, 136)

Following some central results in CT, we can now introduce in precise categorial jargon our syntactic-semantic categories.

a) We can start focussing a mathematical object which resulted central in Caramello’s work: the Alexandroff topology.

Definition 3.2. For P a poset, write $Up(P)$ for the topological space whose underlying set is the underlying set of P and whose open subsets are the upward closed subsets of P : those subsets $U \subset P$ with the property that $(x \in U)$ and $(x \leq y) \Rightarrow (y \in U)$.

This is called the Alexandroff topology on P .

Proposition 3.3. This construction naturally extends to a full and faithful functor.

$$Up : \text{Poset} \rightarrow \text{Top} \rightarrow \text{Locale.}$$

Proposition 3.4. For P a poset, there is a natural equivalence $Sh(Up(P)) \simeq [P, \text{Set}]$ between the category of sheaves on the locale $Up(P)$ and the category of copresheaves on P . (nLab, *Partial Order*)

b) We can define *Loc* (Locales) as a category whose objects are locales (themselves categories) and whose morphisms are continuous maps between them. *Loc* is used to make topology with different objects, possibly lacking (some) points in the subjacent set, in contrast with standard topological spaces.

Loc is naturally a (1,2)-category, where «1» indicates the internal composition of the Locales intended as objects and «2» indicates the maximal dimensions of the directed objects.

c) We can define an elementary topos as a vertical categorification of a Heyting algebra: inside of the topos, the algebras are treated as objects. Heyting algebras map propositions to truth-values; in terms of geometric logic, a Grothendieck topos too can be presented as a such categorification. From the other side, a locale is precisely definable as an (elementary) *Grothendieck (0,1)-topos*, and its categorification in *Loc*, consequently, as a *Grothendieck (1, 2)-topos*.

... And here is our main point: we can define our more basic (intralinguistic and relative to a unique kind of phrase) *Syntactic-Semantic Categories as Grothendieck (0,1)-topoi*, and the maps between the posets and the spine as (1,2)-topoi, i.e. in terms of the *Loc* category, where 2-morphisms are frame-homomorphisms.

Although the existence of *topoi* with an *internal classical logic* is well known, and their internal algebras are indeed defined as *internal boolean algebras*, the *topoi* maintain usually an *internal intuitionistic logic*. (See N-Lab Online, *Heyting Algebra*) Precisely the latter is the *categorical (category-internal) logic* we were talking about. The philosophical and practical consequences of adopting such an internal logic constitute of course a very important argument, which I plan to discuss elsewhere as soon as possible. Here is enough to remember, that the adoption of an intuitionistic logic means to *quantify over existing (given) objects*²⁶ - a condition that in our case (treating pointers and actual cognitive abilities) can be seen more as an advantage than as a handicap.

d) It would be possible now proceed to define higher *Syntactic-Semantic Cats as (n, r)-sites*, by determining first the morphisms between phrase(s) and spine (epi/mono-adjunction), then higher-order morphisms between different spine-phrases in the same language, and finally among the phrasal structures of different languages. The details can be left to a future occasion.

To resume, we can define our conceptual-mathematical path *from semantic to syntax* in this way: we go

- From Posets to Alexandroff Topologies
- From Alexandroff Topologies (Locales) to Grothendieck (0,1)-topoi
- From Grothendieck Topoi to Heyting Algebras
- From Heyting Algebras to Intuitionistic Logic

And reversely, from *syntax to semantics* (we could call it *syntactic path to content*), we can go

- From Posets to Coherent T-Categories
- From Coherent T-Categories to Geometric Theories
- From Geometric Theories to Grothendieck Topoi
- From Grothendieck Topoi to Topological Spaces intended as a semantic dimension (so to say: possible formal worlds)

8. Conclusions

If a theoretical linguist would try to recognize the pivotal element of the whole model, it should result in her evidence that it's the *cartographical representation* that plays that role - it is indeed our threshold to pass from the linguistic basis to i) mathematical formalization (category theory starts with the link *spine-cartographical elements*, intended as an epiadjunction), ii) metaphysical abstraction (the very nature of the linguistic concepts), iii) epistemological foundation (their relation with spinal elements) and iv) neurological representation (cartographical types of information should correspond to the attractors Do y Hasselmo are talking about).

This kind of representation is possibly the key to rescuing that kind of interdisciplinarity that constituted the very engine of novelty (the trigger of all cognitive revolutions, given that there is more than one, literally during the '60s and the '70s (Boccafogli, 2023, 139-140). One of the most attractive tasks for contemporary philosophers of language should be to represent in the proposed terms, that is, on the basis of different conceptions and formalizations of semantics, the

relationship between logic, philosophy of language, and cognitive / bio-linguistics - in search for a comprehensive model and a single concept.

A second point of great interest is that CT represents once more the best way to relate with each other the most disparate / metaphysically problematic domains: in this particular case, neurobiological representation and linguistics are bridged through the isomorphism subsisting between Lambda calculus (see Do & Hasselmo 2021) and Heyting Algebra.

Beside discussing the adoption of an intuitionist logic in a classical general frame (analysis of the structure of a Grothendieck-topos), it remains to be done: a) an analysis of the duality subsisting between intuitionistic logic and modality; b) a precise philosophical and possibly formal definition of hyperintensional differences as source of semantic meaning-changes; c) a more detailed treatment of the Higher Categories; and finally, but maybe most important: d) a compromise between intensional semantics and structured propositions.

Notes

1. See for example Ehresmann & Vanbremeersch, 2007. Do & Hasselmo, 2021, are working on analogous topics, and find many point of convergence with our perspective - as we will see.
2. See for example Bradley, Lewis, Mater, & Theilman, 2018. Or the DisCoCat-interpretation of Coecke, Sadrzadeh & Clark, 2010; Coecke, Bolt, Genovese, Lewis, Marsden & Piedeleu, 2016.: and finally: Coecke, de Felice, Meichanetzidis & Toumi, 2020.
3. See Gómez-Ramírez, 2014.
4. See for example Boccafogli, 2022. The attention conceded to non-human and early infantile conceptualization drives nowadays, under many aspects, the study of the relations subsisting between logic and cognitive sciences.
5. See Do & Hasselmo, 2021: «many AI researchers today are stepping away from pure syntactic representations and emphasizing the importance of tying semantic meaning to symbol (Santoro et al. 2021)».
6. See Harris, 2021 (1993), 269-270: As David Lewis, Montague's philosophy hallmate at UCLA, put it, in an arrow at the heart of the
7. See, for example, the extraordinary book by Moot and Retoré, 2012.
8. Talking about the new wave of philosophy of language emerged during the '60s, Williamson describes in this way the compositional constraint: «Indeed, it was natural to expect a tight relation between the semantic and syntactic structure of an expression, at least at the level of deep structure or logical form. For it was a fundamental tenet of new-wave philosophy of language, coming through Carnap from Gottlob Frege, that the semantics must be compositional, in the sense that the meaning of a complex expression is determined by the meanings of its constituents; how else to explain our ability to understand sentences we have never previously encountered, if made up of familiar words in familiar types of combination? The initial hypothesis must surely be that the requisite semantic articulation of sentences into their semantic constituents matches their syntactic articulation into syntactic constituents at some deep enough level. The compositionality constraint exerted a powerful force in the direction of systematicity» (Williamson 2007 (2022), 329-330). He adds, referring to Montague, that «His work had a major influence on Barbara Hall Partee and has been seminal for a major tradition of intensional semantics as a branch of linguistics (330).
9. This is indeed the focus chosen for the best book I know on the argument, Harris' *Linguistic Wars*, 2021 (1993).

Katz-n-Fodorian program, «semantics with no treatment of truth conditions is not semantics» (1970:18). Montague's work was crafted into an appealing approach for linguists through developments by his UCLA colleagues, principally Lewis and Barbara Partee. The name Montague Grammar—coined by Partee as a commemorative gesture—largely gave way to Montague Semantics in the 1980s, in part because the semantics proved quite portable, getting incorporated into other frameworks, and in part because it never developed the scope of other frameworks. (...) Partee at their head, a small group of linguists enriched the model. Partee drew together Montague's and Lewis's work, and amalgamated it with transformational syntax. Partee, if anything, was an Interpretivist, while Lewis «was quite sympathetic» to Generative Semantics, so the architects represented a kind of détente.

10. Chomsky expressed the Lexicalist principle in his famous *Studies on semantics in generative grammar*, written and presented to linguists' audiences in 1967–1969 and printed in 1972.
11. «If the initial conception of the Principles-and-Parameters approach (Chomsky, 1981) assumed that parameters are expressed on principles of Universal Grammar (e.g., the set of bounding nodes taken into account for Subjacency can vary crosslinguistically to some extent; Rizzi 1982, chapter 2), it was soon realized that this approach needed revisions for various reasons. A prominent problem was that certain crosslinguistically variable properties were linked to the presence of a given item in the lexicon of a particular language, rather than being global properties of a grammatical system. For instance, long-distance anaphora could not be regarded as a global property of the Binding Theory of a particular language, but rather as keyed to the presence of specific items (e.g., Icelandic sig) in the language's lexicon (Manzini & Wexler 1986). This and other considerations (see Rizzi 2014 for discussion) led to what is sometimes called the Borer–Chomsky conjecture (see Borer 1983): (1) The locus of parameters is the functional lexicon». (Cinque & Rizzi 2016, 142-143).
12. In the famous letter of Ross and Lakoff about generative grammar and its semantic dimension, 1967, the authors say: «We believe semantics may be generative».
13. The main and most influent proponents were ironically called «The Four Horsemen of the Apocalypse»: Lakoff, Postal, Ross, and McCawley.
14. See for example Retoré & Signes, 2004; and Lecomte, 2008.
15. «Making a decision requires you to balance multiple constraints, not just applying a simple rule (...). Neural networks provide a computationally efficient way of figuring out how to satisfy multiple constraints. (...) highly distributed neural networks like those in the Semantic Pointer Architecture can carry out parallel constraint satisfaction» (Thagard 2019, 36-37).
16. There's no clear separation of philosophical and logical semantics, given the logic-philosophical nature of the principle of compositionality; furthermore, as seen, linguistic semantics give rise to the new wave of philosopher of language in Anglo-Saxon countries. This convergence-point is exactly where we want to be, and remain for long. The times of Montague and Lewis need to be revitalized, reactualized in nowadays *scientific philosophy*.
17. In *Über Sinn und Bedeutung*, as known, Frege compares the moon with the reference (*Bedeutung*), the telescope with sense (*Sinn* - not differentiable, in principle, from semantic meaning) and the representation of the observer with the internal image (I-semantics of I-language, we would say in a contemporary lexicon).
18. See Williamson, 2024, chapter three: «Case Study: Hyperintensionalism».
19. See Chomsky, 2015 (1995), 28: «lexical entries contain at least some syntactic information, in addition to the phonological and semantic information that surely must be present». And 31: «A typical lexical entry consists of a phonological matrix and other features, among them the categorial features N, V, and so on; and in the case of Ns, Case and agreement features (person, number, gender), henceforth ϕ -features».
20. See for example the (rightly) celebrated Gärdenfors, 2000.
21. It's a very important theme also in decision theory; see for example Fisher, 2022a&b.
22. Williamson, 2024, chapter three: «Case Study: Hyperintensionalism».
23. See Lewis, 1970. In philosophy of language, the normative maxims of assertion witness the importance of this link. Also from an anthropological point of view this relation is very important: true infos were indeed necessary in the logic of groups-survival.
24. «The Universal Base Hypothesis (henceforth UBH) goes back to the early days of generative grammar (Chomsky 1965; Bach 1968; Lakoff 1970; Ross 1970 (1968)), but has been revived and updated over the years. In its early days, the base comprised both lexical rules and phrase structure rules. The former set of rules was responsible for the categorization of words (...) whereas the latter was responsible for word order» (Wiltschko 2014, 10).
25. An active vs. a passive form are not cartographically identical - we are indeed trying to define an *intensional* equivalence.
26. See *Kripke-Joyal semantics* in nLab online. And see Kripke, 1965.

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