**Kevin Picado Lara**

# Space-time functionalism: a guide for the perplexed[1]

**Resumen:** *Según el funcionalismo de marcos inerciales, una estructura jugará el rol de espacio-tiempo en caso de que describa una estructura de marcos inerciales. El presente artículo proporciona una introducción al tema en el contexto de debates más amplios sobre la ontología del espacio-tiempo asumiendo conocimiento previo mínimo.*

**Palabras Clave:** *Simetría, Relatividad, Funcionalismo, Espacio-tiempo, Realismo*

**Abstract:** *According to inertial frame functionalism, a structure will play the space-time role just in case it describes a structure of inertial frames. The present paper provides an introduction to the topic in the context of wider debates about the ontology of space-time assuming a minimal technical background.*

**Keywords:** *Symmetry, Relativity, Functionalism, Space-time, Realism*

## 1. Symmetry sings its siren song

To even begin discussing what a space-time is or might be, we must first learn what a symmetry is, and to do so, we must first learn the basics of its language: group theory.

A group $G$ is a non-empty set equipped with a binary operation that associates any two elements $g, h$ the product $g * h$ such that

1. There is a neutral element or «identity» $e$ such that $g * e = e * g = g$.
2. Each element $g$ must have an inverse $g^{-1}$ such that $g * g^{-1} = g^{-1} * g = e$.
3. It's closed under $*$ such that $g * h = k$, $\forall g, h, k \in G$
4. $*$ is associative such that $g * (h * k) = (g * h) * k$, $\forall g, h, k \in G$
5. If $g * h = h * g$, $\forall g, h \in G$ the group will be called abelian
6. Groups can be finite or infinite

The set of all integers $\mathbb{Z}$, with $*$ defined as addition, for example, is an abelian infinite group.

You can make these groups act on other objects to *transform* them. If the members of your group are rotations and you make it act on a triangle, the triangle will rotate by a certain amount. Structure-preserving transformations also form a group. The neutral element is the identity transformation, which simply transforms any object into itself. Think of a 90° rotation of a square or rotating a sphere in general. The group of transformations that leave an object unchanged or invariant is called the symmetry of the object.

This is all fairly abstract: a group is a set of elements and how they are mixed together, without the need to specify which objects make up the group. But we can make it more concrete with the help of group representation theory. A representation is, roughly speaking, a way of associating elements of the symmetry groups with mathematical objects with which to work, e.g. numbers, matrices, etc. In this way, the group of rotations about a given point in 3D space, referred to as SO (3), can be visualized by multiplying by a 3x3 matrix that affects the components of the rotating vectors. This matrix *represents* the rotation.

Even though the study of group theory proper arose only around the turn of the 19th century with Galois' study of the solutions for polynomial equations of degree greater than four, by the 1830's J.F.C Hessels, working in the field of crystallography, had already made use of symmetries in the finite subgroups of the 3D rotation-reflection group for the identification of different classes of crystals, and in 1884 Pierre Curie published *Sur la symétrie*, where he discussed the close relationship between physical properties of matter and the type of symmetry of the medium.

Symmetry principles made their appearance in 20th-century physics in 1908 with Hermann Murkowski's identification of the space-time invariance group, and in 1918 Emmy Noether published her two now famous theorems on group invariance in variational problems, according to which there is a fundamental connection between the symmetries of a physical system and it conservation laws. But widespread acceptance of this field in physics would have to wait until the work of Hermann Weyl and Eugene P. Wigner in the late 1920's and early 1930's, respectively.

It was Weyl himself and his failed 1918-9's attempt (partly inspired by Husserl) to create a «pure infinitesimal geometry» capable of unifying the gravitational picture given by general relativity (GR) with electromagnetism that laid the ground for the correct understanding of what was to become gauge theory.

Gauge theories are, very roughly speaking, a class of theories based on the assumption that certain symmetries are possible not only globally, but also locally. That is, that it's possible to carry out these symmetry transformations only in a particular and limited region of space-time without affecting the rest of the universe. Maxwell's electrodynamics, quantum electrodynamics, quantum chromodynamics, and the standard model are all gauge theories.

GR can also be treated (somewhat polemically) as a gauge theory, where its gauge invariance corresponds to the invariance under transformations of the diffeomorphism group. This is related to the invariance under coordinate transformation of the equations of GR, which grants us freedom to choose our coordinates, referred to as the gauge freedom of GR. Its relevance to debates on the ontology of space-time, the substantival-relationalist debat[2], was the use of this freedom to formulate the now (in) famous hole argument (first outlined by Einstein between 1913- 1915). The hole argument was intended to show that in a scenario where a gauge transformation takes us from a state of affairs $\phi(x)$ at a time $t$, to an observationally indistinguishable but mathematically distinct state of affairs $\phi'(x)$ at a time $t'$, substantivalism—the view that space-time is a collection of events (points at a specific time and location) with independent existence—corners itself into a position where, because of its metaphysical commitments, it must argue $\phi(x)$ and $\phi'(x)$ are different in a sense not covered either by the theory or by observation, while relationism—the view that considers the relations between events as primitive and denies spatio-temporal points any kind of robust existence— can argue this is a purely mathematical difference not reflective of physical reality (Earman and Norton 1987).

In the decades that followed, a lot of ink was spilled with no resolution in sight; can it be avoided? Does it really leave relationism unscathed? Does interpreting gauge invariance this way even make sense? Each passing year filling John Earman's (1989) words with more relevance:

> My own tentative conclusion from this unsatisfactory situation is that when the smoke

of battle finally clears, what will emerge is a conception of space-time that fits neither traditional relationism nor traditional substantivalism. (Earman 1989, 208)

In more recent years, however, an arena of discussion has open up in connection with the various research programmes on quantum gravity, an idea almost as old as quantum mechanics but of relatively recent boom in the philosophy of physics[3], especially around a result present in an almost generic way across these programs: the possible disappearance of space-time at the fundamental level. If such non-spatiotemporality were to be confirmed, it is feared (e.g. Huggett and Wüthrich 2013), many theories about quantum gravity, and their consequences to the substantival-relationalist debate, would be rendered empirically incoherent.

In response to these fears, the idea of space-time as a functional concept has emerged as a possible way of avoiding disaster. In short, it is argued that a definition of space-time can be given in terms of its functional role, where such a role can be played by some entity (or set of entities) in the theory. But a full understanding of what is really meant by this will require us to go on a little interpretational trip.

## 2. Old with a new coat of paint

In the opening discussion of his monumental history of dynamics, Barbour (2001) reminds us that every change in our conception of motion has been equivalent to a change in our deepest conceptions of things, «each change in our concept of motion opens the door into a new world» (Barbour 2001, 1). One way of framing this, following Earman (1989), is to ask ourselves, what questions about motion are meaningful in what space-time? For example, to say that the questions «is this particle moving? » and «how fast is it moving? » are meaningful is to claim that there is a preferred way of identifying spatial locations through time, constraining our choice of space-time to those equipped with a notion of preferred inertial reference frames (e.g. Newtonian space-time), and tossing aside those that aren't (e.g. Leibnizian space-time).

Note the use of «space-time» instead of «space» or «space and time». Since the beginning of the 20th century many physicists, mathematicians and philosophers have made the interesting observation that any physical theory, not just special relativity (SR) and GR, can be formulated in the four-dimensional spacetime framework given by Minkowski. This framework allows for the reconstruction of the space-time structure assumed in classical physics. Such reconstructions equip us with the right tools to explore and classify the different space-times based on their structural «richness». Wallace (2019), following Klein's Erlangen program—a method of studying geometric structures in terms of certain transformation groups which preserve elementary properties of the given geometry—provides us with the following list of classical space-times going from lowest to highest in terms of structure:

1. Machian spacetime
$t \to f(t)$ for monotonic f, $x^i \to R^i_j x^j(t) + a^i(t)$

→ Absolute simultaneity + euclidean metric = invariant relative distance

2. Leibnizian space-time
$t \to \pm t + \tau, x^i \to R^i_j x^j(t) + a^i(t)$

→ Mach + time metric = invariant relative velocity and acceleration

3. Maxwellian space-time
$t \to \pm t + \tau, x^i \to R^i_j x^j + a^i(t)$

→ Leibniz + a standard of (non)rotation = invariant rotation of bodies / difference between linear and rotational motion

4. Galilean or neo-Newtonian space-time
$t \to \pm t + \tau, x^i \to R^i_j x^j + v^i t + a^i$

→ Maxwell + inertial structure = invariant absolute acceleration

5. Newtonian space-time
$t \to \pm t + \tau, x^i \to R^i_j x^j + a^i$

→ Galileo + absolute space = invariant absolute velocity

Where $R$ is a matrix, $v$, $x$ and $a$ are vectors and $t$ a constant. The mathematical expressions associated with each space-time correspond to the transformation rules given by their symmetry groups (e.g. 4 is just a fancy way of writing the Galilean transformations you might encounter in an introductory physics class). These rules do not indicate mere coordinate transformations, but point to maps of the kind $(x^{\alpha}, t) \rightarrow (x^{\alpha\prime}, t')$ that preserve the entire structure of space-time. They should not be understood as changes from old to new coordinates of the same points, but as transformations that take us from an old point $(x^{\alpha}, t)$ to a new point whose coordinates in the old coordinate system are$(x^{\alpha\prime}, t')$ (Earman, 1989 p.41). The most well-known example is in SR, where the transformation rules are given by the Lorentz transformations encoded in the Lorentz group.

Mathematically, we can say that a space-time is a collection of four-dimensional points (events) plus some additional structure, typically one or more metrics $g_{ab}$ (also called metric tensor), represented by a $n \times n$ matrix, that allows us to measure spatial and temporal distances between points. Different ways of specifying distances between points produce different types of space-times. We can identify at least two:

- Classical space-times, where spatial and temporal distances are absolute and there is a separate spatial and temporal metric.
- Relativistic spacetimes, which have a single spatio-temporal metric, and its division into spatial and temporal parts depends on the inertial reference frame of the observer (= spatial and temporal distances are relative to the observer).

Metrics can also be flat or curved: how the distance between points is specified encodes the curvature of spacetime. Classical spacetimes can be flat (Newtonian space-time) or curved (Newton-Cartan space-time), just as relativistic spacetimes can be flat (Minkowski space-time) or curved (general relativistic space-times).

And just because a space-time is classical it does not mean that its internal structure will be less complex than a relativistic one. Consider a four-dimensional space-time labeled by $(ct, x, y, z)$ and the principle of relativity—the requirement that law-governing equations remain invariant for all and any inertial frames—encoded by $\Lambda^T g_{ab} \Lambda = g_{ab}$, where $\Lambda$ is a transformation matrix and $\Lambda^T$ its transpose:

- If we're working with SR, the metric is defined as minkowskian, $g_{ab} = \eta_{ab}$ with $diag(-1, 1, 1, 1)$. This make it so $\Lambda$ must be made up of 3 rotations and 3 boosts (the Lorentz transformations) in order for $\Lambda^T \eta_{ab} \Lambda = \eta_{ab}$ to hold, returning a relatively simple flat four-dimensional Minkowski space-time $\mathbb{M}^4$.
- If we're working with Galilean relativity, $\Lambda$ must also be made up of 3 rotations and 3 boosts, but we'll have two metrics instead of one, $g_{ab}$ with $diag(1, 0, 0, 0)$ and $\tilde{g}_{ab}$ with $diag(0, 1, 1, 1)$: the Galilean metric is degenerate (i.e. its determinant is zero), so it's necessary to provide an additional metric to measure spatial separations. The resulting Galilean space-time will be described as a fiber-bundle with base space $\mathbb{R}^1$(time) and fiber $\mathbb{R}^3$ (space).

Similarly, determining what spatio-temporal structure is implied by classical theories of space is not a simple or controversy-free topic. In particular, which space-time correctly accommodates Newtonian mechanics as described by Newton has been the subject of much discussion.

Although it was generally assumed that to accommodate Newtonian physics, one would need, at a minimum, to pose an intermediate space-time between the Leibnizian and Newtonian, the Galilean space-time[4], more recent debates have cast doubt on the success of such a project, especially regarding the interpretation of Corollary VI to the Principia:

> Corollary VI. If bodies, anyhow moved among themselves, are urged in the direction of parallel lines by equal accelerative forces; they will all continue to move among themselves, after the same manner as if they had been urged by no such forces.

In other words, a system of bodies experiencing uniform linear acceleration looks no different from a non-accelerating one. This introduces extra symmetries (e.g. a symmetry under linear acceleration) not covered by neo-Newtonian space-times, how do we include them?

Eleanor Knox (2011, 2013a) suggests this can be done by moving to a modified version of Newtonian gravity (NG) known as Newton-Cartan gravity (NCG). This ensures its empirical equivalence with NG while allowing her to reap the benefits of geometrizing away certain underdetermination about the choice of gravitational potentials present in the mathematics when the full group of symmetries is considered.

Although NG retains its empirical consequences, in this theory, as in GR, gravitation is not conceived of as a force; instead, it is a manifestation of space-time curvature: space-time is curved by the distribution of matter in the universe, and the motion of bodies in space-time is influenced by that curvature. Knox (2011) sets out to defend this view by saying:

> Philosophers and physicists have long known that general relativity's uniqueness does not lie in its mathematical format alone: [NG] can also be written in the language of differential geometry. Moreover, it may be reformulated in this language in such a way that [NG], as in GR, appears to be a manifestation of geometrical spacetime structure. (...) One obvious worry is that we have here physical examples of the conventionalist thesis: it seems we must accept that the geometry of spacetime is underdetermined by data or else accept that it is not an objective feature of the world. I argue here that such a conclusion is not warranted; the full structure of our complete set of physical theories and the data they entail is enough to choose between geometries. This is because the concept of an inertial frame is both more central, and more robust, than the literature typically gives it credit for. (Knox 2011, 1-2)

The role given here to inertial frames will be central to our story. In arguing in favor of NCG against claims of underdetermination, Knox reaches the following conclusion:

> Careful consideration of inertial structure revealed that geometrical form does not always determine a theory's spatiotemporal commitments (...) We see that geometrical form is not a sufficient condition for representing spacetime structure. Inertial considerations play an important role in the process by which mathematical structure comes by its spatiotemporal credentials. (Knox 2011, 11)

And in a more explicit discussion of the role of inertial frames in NG she notes that:

> In the Newtonian context, where no spacetime metric exists [spacetime structure is not represented by a metric field, but by 2 metrics and a derived covariant operator], the sole role of spacetime structure (as opposed to spatial or temporal structure) is to represent the structure of inertial frames. As a result, the connection associated with the inertial frames is the one that represents spacetime structure. (Knox, 2013a, 11)

Application of this philosophy to the concept of spacetime in general will result in the position now known as inertial frame functionalism. Her maxim then is, that the role of space-time will be played by anything just in case it defines a structure of local inertial frames.

The foundations of this thesis will find their home in Brown and Pooley's (2001, 2005, 2006) dynamic interpretation of SR, based on the supposed operational importance of the structures assumed as spatio-temporal in our physical theories. How all this translates the SR and GR, as well as its shortcomings and alternative formulations, will be the subject of the following sections.

## 3. It's all symmetry once again

The notion of functionalism used in the literature is greatly indebted to its philosophy of mind counterpart, according to which what makes something a particular state of mind does not depend on its internal constitution, but on the way in which it *functions*, or said another way, the role it plays in the system of which it is a part. Thus, for example, mental states can be multiply realizable, i.e. it is possible that the mental state of being in pain present in us could also be present in an alien with a different physical constitution (an equifinality of sorts).

At the same time, it is also a form of *in re* structuralism à la Shapiro (1997), «inasmuch as functionalism about a property involves identifying that property with a place in a structure» (Knox forthcoming, 3).

In the physical setting, then, the functional role of a physical entity or structure is its role in physical laws, which often boils down to its implications for material objects. This in turn motivates the reading of Brown and Pooley's dynamical approach to relativity as an argument for inertial frame functionalism. According to this interpretation, relativistic kinematic phenomena such as length contraction and time dilation, as well as inertial motion and the geometry of Minkowski space-time, are explained by the fact that the dynamical laws governing the behavior of material objects are Lorentz covariant, i.e. because of the fact that these equations retain their validity under Lorentz transformations.

Going back to classical spaces, we might characterize this approach as promoting the idea that the inertial structures of classical space-times are what they are because of what Newton's laws imply about the behavior of matter, instead of matter behaving a certain way because of the space-time geometry.

This goes against what is known as the geometric explanation, developed explicitly by authors like Janssen (2002a, 2002b, 2009) and Maudlin (2012) but more or less assumed by the general physical literature. The difference between these positions can be characterized in terms of the direction of their arrow of explanation (Sus 2020) in answering the question of why, in SR, do the dynamical symmetries (i.e. symmetries of the equations of motions) coincide with the symmetries of the Minkowski metric $\eta_{ab}$, what explains what, what reduces to what? Janssen and Maudlin say that the arrow of explanation goes from the metric to the symmetries of the laws, the metric as the explanandum with the symmetries as the explanans, while Brown and Pooley think that it goes from the symmetries of the laws to the metric, symmetries as the explanandum with the metric as the explanans. $\eta_{ab}$ is taken to be either ontologically autonomous (metric substantivalism) or just a mere encoding of the symmetries of the laws[5].

As for the geometric explanation, Brown (2005) says that:

> It is doubtful at best whether the geometries of the configuration space in classical mechanics, or the space of equilibrium states in thermodynamics, play the kind of explanatory role that the spacetime interpretation of SR attributes to Minkowski geometry. Why should space-time geometry be any different? It might be thought that space and time are somehow more fundamentally physical than the other space, or more accessible to the senses, or that they combine to form the arena of physical events. In short, they are more real. But is not this reasoning question-begging? (Brown 2005, 156)

What's wrong with the geometric explanation? In the opinion of Knox (2013b, forthcoming) and Brown (2005), the geometric explanation is not (by itself) a good way to identify spatiotemporal structure, except to the extent that geometric objects prove they can play the space-time role. But all kinds of geometric objects end up having nothing to do with space-time, for example:

> If the bare, differential space-time manifold [the arena where events «take place»] is a real entity, then different solutions of Einstein's field equations that are related by diffeomorphisms correspond to different physical states of affairs [i.e. the hole argument]. The theory is incapable of predicting which of the different possible worlds is realized, but all of them are, as we have seen, empirically indistinguishable. The simplest (...) conclusion (...) is that the spacetime manifold is a non-entity. In this case the different, diffeomorphically related worlds are not only observationally indistinguishable, they are one and the same thing. (Brown 2005, 139)

> The metric field in general relativity, for example, turns out to be just the right kind

> of thing to describe a variably curved 3+1 dimensional space; isn't it just obvious that it represents spacetime? But on further reflection it is still more obvious that geometrical considerations alone don't pick out spacetime structure. The metric field is a rank-2 metric tensor field of Lorentzian signature. But this feature is neither necessary nor sufficient to represent spacetime. (Knox, forthcoming, 16)

The basic tenet of the dynamical approach, then, is that the features of space-time are to be fundamentally understood as features of the dynamical laws. The geometry of Minkowski space-time is parasitic on the relativistic properties of dynamic matter fields and is nothing more than an encoding of the behavior of bodies, i.e an encoding of the symmetries of matter fields:

> Relativistic phenomena like length contraction and time dilation are in the last analysis the result of structural properties of the quantum theory of matter (...) Lorentz contraction is the result of a structural property of the forces responsible for the microstructure of matter. (Brown 2005, vii-viii)

> If it is the structure of the background spacetime that accounts for [these phenomena], by what mechanism is the rod or clock informed as to what this structure is? How does this material object get to know which type of space-time—Galilean or Minkowskian, say—it is immersed in? (Brown 2005, 8)

This position will be then extended to the GR context:

> Despite the fact that in GR one is led to attribute an independent real existence to the metric field, the general relativistic explanation of length contraction and time dilation is simply the dynamical one we have urged in the context of special relativity. (Brown and Pooley 2001, 271)

This movement is complicated, since the dynamic metric field in GR cannot be easily reduced to matter fields. This change in metrics can be illustrated as a movement from a metric with $diag(-1,0,0,0)$ in SR that's valid everywhere, to a more general metric field with $diag(a,b,c,d)$ in GR, where its exact form varies depending on its «location» since curvature prevents you from having a global rule to define distances. From here onwards I'll use $g_{\mu\nu}$ to distinguish it from other metrics, and refer to it as a Lorentzian metric.

Up to this point, there is no real difference between the geometric and dynamic approaches, as we move to GR with respect to the ontological state of the metric (Read, forthcoming): both the dynamic and geometric approaches must take the metric field to be an autonomous entity with its own set of dynamical laws, the Einstein's field equations (EFE).

The divergence occurs only when considering its chronogeometric importance (the link between the proper time registered by an ideal clock and the metric). Here «geometers» will say that the metric field $g_{\mu\nu}$ constrains the possible forms of the dynamical equations for matter, so that the metric symmetries coincide locally with the dynamic symmetries, while the «dynamicists» will argue that

> We need not interpret $g_{\mu\nu}$ as a metric (...) [EFE] do not rest on such an interpretation (...) it is only the geodesic motion of massive particles that can be read (...) off from the general form of the [EFE]. (Brown 2005, 160)

Acquiring its chronogeometric importance only through the Strong Equivalence Principle (SEP):

> A possible spacetime, or metric field, corresponds to a solution of the [EFE], but nothing in the form of the equation determines either the metric's signature or its operational significance (...) It is a component of the [SEP] that in 'small enough' regions of space-time the physics of the non-gravitational interactions takes its usual [Minkowskian] form (...) From the perspective of the local freely falling frames, [SR] holds when the effects of space-time curvature (...) can be ignored. It is this extra assumption, which brings in quantum physics. (Brown 2005, 9).

The SEP expresses precisely that feature of the relationship between matter fields and the metric which ensures that systems built with these matter fields reflect the geometry of the metric field. Space-time is nothing but the matter fields and their properties, and the relations among them.

Knox will try to give it a more realist reading and say that any structure equipped with this kind of chronogeometric importance can be identified as playing the functional role of space-time.

## 4. Inertial frame functionalism

Knox (2013b) begins her argument by giving us three conditions needed to identify something as an inertial frame in SR and NG:

1. Inertial frames are frames with respect to which bodies free of forces move with constant velocities.
2. The laws of physics take the same simple form in all inertial frames
3. All bodies and physical laws select the same equivalence class of inertial frames

The challenge now is to find something that satisfies these requirements in the context of GR such that it can play the role of space-time. Brown (2005) notes that:

> A more far-reaching claim is the [SEP], which will be defined here as follows. There exist in the neighborhood of each event, preferred coordinates, each called locally inertial at that event. For each fundamental non-gravitational interaction, to the extent that tidal gravitational effects can be ignored, the laws governing the interaction find their simplest form in these coordinates. (Brown 2005,169)

The SEP refers to the fact that in GR, unlike in SR or NG, inertial frames are well defined only in an infinitesimal neighborhood of a space-time point. Moreover, unlike the second condition, «[GR] does not possess laws that take their simplest form in inertial coordinates» (Knox 2013b, 3), so its application is limited to laws referring to some coordinate system. This highly constrains what can count as an inertial frame. How, then, are we to identify Knox's inertial frames? The trick is to define them with the help of the tetrad formalism.

In GR, a tetrad (also called a field of tetrads) is a set of four orthonormal vectors, one temporal and three spatial, defined on a Lorentzian manifold, i.e. a manifold equipped with a Lorentzian metric. While in standard GR we take components of the metric with respect to a coordinate basis, in the tetrad formalism we take them with respect to the orthonormal basis formed by the above vectors.

Every event $p$ on some observer's worldline (the observer's 4D path across space-time) $x^{\mu}(\tau)$ in the associated space-time has a space triad that that observer carries with it. These spatial vectors may then be taken as defining the spatial coordinate axes of a local laboratory carried along with the observer. Observations are then made with respect to the axes and the clock («ticking the time» with respect to the time vector) of this laboratory, since they form an orthonormal basis on which the observer is at rest. These axes form a local inertial frame at each point, and the dot product of these vectors will satisfy the Minkowski metric. Knox (2013b) further narrows the definition:

> A tetrad field is holonomic on a neighborhood just in case it's possible to define associated coordinates everywhere on the neighborhood. Physical reference frames can be assigned coordinates, so we'll require that a tetrad field be holonomic in order to represent a physical reference frame. (Knox 2013b, 349)

To go from a reference frame to an inertial reference frame in a flat space-time we just need to construct our tetrad field such that:

1. The coefficients of the connection $\Gamma^{i}_{jk}$ vanish with respect to the reference frame. The connection is an object usually introduced as an ingredient in the definition of the covariant derivative, parallel transport,

the geodesic equation, the Riemann tensor, etc.; its physical importance lies mainly in its role in curvature «detection» and its relation to the metric (different metrics usually return different connections), for example, if the metric is flat then $\Gamma^i_{jk} = 0$ and its coefficients are said to vanish (given a suitable coordinate system).

2. The metric takes the form of the Minkowski metric $\eta_{ab}$ with respect to the reference frame.

But as expected, things will get more complicated in GR since we are limited to local inertial frames. To find them, we'll limit ourselves to the neighborhood of around a point on the manifold:

> For in a GR spacetime we can if we wish define a holonomic tetrad field on N [the reference frame] that is normal and orthonormal along a given curve, although it won't generally be normal or orthonormal elsewhere in N off the curve. I'll call this a locally normal frame. The coordinates associated with such a tetrad field are Fermi coordinates, normal and orthonormal along the curve, but generally not elsewhere in the neighborhood. (Knox 2013b, 4)

By *curve* Knox means a geodesic. The basic idea is that things move along the most «straight» paths possible. These can be defined as the paths that extremize the distance between two points and are called geodesics. In three-dimensional Euclidean geometry, a straight line is defined as the shortest distance between two points, and Newton's 1st law says that in the absence of external forces, particles move along such lines. It's a similar story in GR, but sometimes it's the «maximum» interval that's relevant, not the shortest. That's why we say it «extremizes» the distance instead of minimizing it. The geodesic equation describing these paths is

$$\frac{d^2x^i}{d\tau^2} + \Gamma^i_{jk}\frac{d^2x^j}{d\tau}\frac{d^2x^k}{d\tau} = 0 \quad (1)$$

Where $\Gamma^i_{jk}$ can be read as representing a «force» due to gravity, which curves a particle's path through space-time, and $\Gamma^i_{jk} = 0$ as the absence of gravity. The expression «curved space-time tells matter how to move» can then be interpreted as:

$$\text{Metric } g_{\mu\nu} \rightarrow \text{Connection } \Gamma^i_{jk} \rightarrow$$
$$\text{Geodesic equation } \rightarrow \text{World-line } x^\mu(\tau)$$

What is relevant here is that in GR inertial frames can be identified with freely falling reference frames, which follow a geodesic. In these frames, gravity will seem to disappear (for a Newtonian equivalent, recall corollary VI). This is because in GR we can reduce gravity to inertia (not establish an equivalence between the two as it's sometimes believed) in our definition of inertial frames.

Knox (2013b) continues:

> In order to ensure this [that the inertial frame follows a geodesic] let's restrict our attention to those holonomic tetrad fields (and associated Fermi coordinates) that are normal and orthonormal on a geodesic. Such tetrad fields now have the right features locally to represent inertial frames, inasmuch as they approximate the features of tetrad fields representing global inertial frames within a small neighborhood of the geodesic
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> In [GR] it's the job of the [SEP] to ensure that the locally normal frames and coordinates defined with respect to the metric do indeed link to the rest of our physics in the right way to ensure that tetrad fields with the right features locally play the role of inertial frames in our theory. (Knox 2013, 349)

In short, in GR the inertial frames are associated with normal Fermi coordinates along geodesics. At the origin of these coordinates, the coefficients of the connection $\Gamma^i_{jk}$ vanish and the metric resembles the Minkowski metric.

Here already we can see a clue to the functionalist strategy: the metric *is* space-time because of what it does (thanks to the SEP) and not for what it is. And it is thanks to this that we can finally make the leap towards inertial frame functionalism, since by defining a structure of local inertial frames in the way described by the SEP, the metric manages to satisfy the set of desiderata for the role of space-time. This is a middle ground of sorts, wherein we arrive at a metric substantivalism via a relationist approach. Knox (2019) justifies this reading on the basis that:

> The local coupling ensures that the local symmetries of the dynamics coincide with the local symmetries of the metric [the so-called Earman prescription, see Earman 1989, 49], and hence ensure that the metric governs the behavior of rods and clocks which obey those dynamical laws. (...) Both the Minkowski metric and the affine structure of Newtonian theories serve to define a structure of inertial frames (...) Moreover, in relativistic theories, inertial structure fixes projective and conformal structure, and hence metrical structure, so the definition does justice to the full geometrical significance of the theory. (Knox 2019, 12)

## 5. Pros and Cons

A number of objections have been raised against inertial frame functionalism (but not necessarily against a functional identification of space-time), the strongest of which is provided by Baker (2020). The supposed weaknesses of the position can be divided into three:

1. A functionalism that takes space-time to be a cluster concept is preferable to an inertial frames functionalism because the criterion of playing the role of inertial structure can neither be necessary nor sufficient to call said structure space-time.
2. It depends on a prior assumption about which coordinate systems defined in a theory are frames of reference, and thus on assumptions about which geometric structures are spatio-temporal.
3. There are theories that lack the kind of inertial structure required by Knox, but include structures that play the role of space-time in a meaningful sense, and theories with space-time structures that will not count as space-time according to Knox, because they do not move towards determining inertial structure.

1 is the least problematic, as Baker himself points out. From conversations held with Knox, he clarifies that she only claims validity for a certain sector of space-time theories (Newtonian and relativistic), in other words she «does not intend to advance her inertial functionalism as a set of necessary and sufficient conditions that any structure whatsoever must meet in order to count as spatiotemporal» (Baker 2020, 2).

Knox (2018) agrees that a notion of theoretical concepts capable of accommodating disagreement and theory-change is necessary, but argues that the cluster concept has not been developed or linked to the relevant literature strongly enough to play such a role (advocating instead alternatives like Wilson, 2006). Furthermore, the cluster concept used by Baker leaves space open for fundamentality, which has to be rejected or rewritten in functionalist terms.

Addressing 3, Baker mentions topological quantum field theories as a «realistic counterexample» (Baker 2020, 9) of theories in which there is a meaningful notion of space-time but in which an inertial structure is nowhere to be found. But he admits that these kinds of theories «exist fairly far outside Knox's intended domain of familiar Newtonian and relativistic physics. So we have not yet ruled out the possibility that inertial functionalism suffices for this class of theories» (Baker 2020, 11).

Regarding the claim that there are a number of obvious examples in which the space-time structure cannot be accommodated by inertial frame functionalism, Baker mentions the orientation of parity or handedness since

> A parity transformation, which mirror-reflects everything in spacetime across some spatial plane, induces no change in which trajectories count as inertial. (...) Since they leave inertial structure invariant, Knox's inertial functionalism would predict that parity and time-reversal must always be symmetries of spacetime. But this is not so.

> In the Standard Model (...) the weak interaction violates parity [see the 1956's Wu experiment] and is also thought to violate time-reversal invariance (...) This means that in the spacetime where weak interactions take place, there must be spatiotemporal structures that determine a preferred direction in time and a preferred parity orientation. (Baker 2020, 11)

Baker assumes that the notion of inertial frames is exhausted by inertial trajectories, which are unaffected by the orientation field, but this is not how an inertial frame functionalist would see it since the uniformity of laws makes up an important part of the definition.

Knox (2018) considers the possibility of such an orientation field inducing a sense of laterality or handedness throughout space-time, as the most plausible counterexample, describing it as a field of tetrads on the manifold that defines a preferred notion of handedness.

In other words, certain phenomena, and the physics behind them, will not be symmetric under parity transformations. A local explanation of such phenomena would call for an operative definition of left and right that makes no reference to other objects or processes. Introducing an orientation field would allow us to do just that (although its introduction is controversial).

She agrees that if we accept its existence (or at least its possibility), then we must count it as a piece of space-time structure, but denies that inertial frame functionalism cannot accommodate it, since there's nothing preventing us from accepting an orientation field as a piece of space-time so long as it plays a role in defining the inertial frame structure. And although it's true that, in a parity-violating universe, this field would not have the necessary structural richness to play the role by itself, it could if we add to it a metric and a connection.

In the Standard Model, electroweak theory, responsible for the prediction of the W and Z bosons, explains the parity violation as a result of the fact that the theory treats the left and right chiral components of the same Dirac field in a different way. In particular, right-handed particle fields do not couple to W bosons at all (Pooley 2003).

Knox (2018) points out that this distinction between right and left is made with reference to the laterality of the Cartesian coordinate system, which means the laws that support the interaction take a uniform form in the right coordinates associated with a particular class of inertial frames. And once the coordinate form of the laws is fixed, a set of inertial frames of particular laterality can be selected. The function of the orientation field will be precisely to choose a preferred class of tetrad fields and their associated coordinates: it selects a set of inertial frames.

The strongest objection will then be 2, the refutation of which seems unlikely. Baker illustrates it by saying that:

> The inertial functionalist is presumably not saying that in a quantum theory (for example), coordinate systems on the Hilbert space of states are candidates for counting as inertial frames (...) Prior to determining which structures are spatiotemporal (which is supposed to be the task of her theory), what right does Knox have to assume that coordinate systems on Hilbert space are not frames? Why not suppose that the inertial frames are the Hilbert space coordinate systems in which the laws take on a particularly simple form, and conclude that spacetime is given by geometric structures on Hilbert space? (Baker 2020, 6)

Here it could be argued that the Hilbert space of states supervenes on space-time. But Baker continues with another example from classical electrodynamics:

> Suppose we hand an inertial functionalist the fiber bundle version of special relativistic electrodynamics and ask her to determine the theory's spacetime structure. Will she give the canonical answer, that the spacetime of the theory is Minkowski spacetime? It depends on which coordinate systems we identify as the theory's reference frames! (Baker 2020, 6)

It follows then that, to the extent that inertial frame functionalism requires both a theory and a specification of reference frames as its input, it cannot give an unconditional answer about the spacetime structure of a theory. It can only provide a conditional answer, of the following sort: «If the reference frames are coordinate systems on the base space, then spacetime is Minkowski spacetime». (Baker 2020, 7)

But this is not as bad as it might seem. Knox (2018) describes herself as committed to the *usefulness* of inertial frame functionalism, something exemplified by her commitment to capture operational spacetime:

> Considering the inertial structure provides a shortcut that allows us to glean the empirical consequences of a theory without going into the messy details of our various measuring devices. (Knox 2018, 347)

Which is why it is still possible for her to maintain her interpretation and reject Baker's cluster functionalism.

In their discussion of the limitations of inertial frame functionalism, Read and Menon (2019) agree with this, noting that although Baker's cluster approach might be correct in that our pre-theoretical concept of space-time cannot be analyzed through an unambiguous set of necessary and sufficient conditions, the complexity of his analysis makes the project lack applicability: «while Baker is morally right on the nature of spacetime, his analysis has limited practical value» (Read and Menon 2019, 18). While Knox's approach, despite not fully capturing the notion of spatio-temporality, can be used to do interpretive work in novel settings:

> Knox gives a simple, functional characterisation of spatiotemporality, which is readily applied to new spacetime theories (...) Knox's analysis has the virtue of readily applicability to new cases. Insofar as one takes inertial frame structure to be a guide to the other qualities which feature in the spacetime concept (...) one may continue to be justified in following Knox's approach. (Read and Menon 2019, 18-19)

The most novel of such settings is the notion of spatio-temporality in quantum gravity. Let's take the case of non-commutative geometry (NCG) (following Knox 2017, Huggett 2018 and Huggett et al. 2020), which in short consists in translating the tools of Riemannian geometry to the Hilbert space formalism of quantum mechanics, and see how it fares.

## 6. Non-commutative space-times

The most salient feature of NCG is the fact that space-time coordinates are taken to be mutually incompatible. This is analogous to the Heisenberg uncertainty relations in regular quantum mechanics and the failure of observables, like position and momentum $[\hat{X}, \hat{P}]$, to commute. To capture this «pointlessness» of spacetime, an abstract algebra of non-commutative coordinates is introduced as a deformation of the ordinary commutative structure of spacetime: instead of spatial points and their relationships, we have elements of an algebra and their relations.

How can space-time arise from a theory of non-spatial degrees of freedom? On the Planck scale, classical Minkowski spacetime (as a commutative algebra) is quantized and described by a non-commutative algebra, i.e. $x_\mu \rightarrow \hat{x}_\mu$. In this way, we van arrive at a non-commutative model of quantum spacetime:

$$[x_\mu x_\nu] = 0 \rightarrow [\hat{x}_\mu \hat{x}_\nu] \neq 0 \qquad (2)$$

Where, for example, the so-called Moyal-Weyl ($\theta$) space-time $\mathcal{A}^\theta$ can be obtained, which is described by

$$[\hat{x}^\mu, \hat{x}^\nu] = ih\theta^{\mu\nu} \qquad (3)$$

where the particular NCG is defined by placing constraints on $\theta^{\mu\nu}$, a constant real-valued skew-symmetric matrix.

The Lagrangian—a scalar quantity that helps us distinguish physical from unphysical motion—that characterizes any theory can be fully represented in algebraic terms, and so, we can have physics in an NCG: a NC field theory. Furthermore, pillars of modern physics, such as Noether's theorems, also survive.

When giving an interpretation, however, the situation is complicated. Once we arrive at an algebraic formulation of differential geometry, we will find a dynamic involving the Moyal star product. One consequence of this will be that the theory does not reflect the diffeomorphism invariance: $\theta$ will not be invariant under ordinary coordinate transformations, being constant only with respect to a privileged class of frames.

But if we take non-commutativity as conducive to curvature and torsion in the induced geometry we will see that the coordinates at which it is constant are the normal Fermi coordinates of the induced metric (Ćirić, Nikolić and Radovanović, 2016)!

And although we will find different geometries (such as a non-commutative fundamental algebra and its structures or an induced affine geometry in the commutative space), we'll be able to choose the one selected by the preferred reference frames of the fundamental dynamics once when it's mapped back to commutative space, which in this case, is the metric geometry.

We can see a clear privileged structure of inertial frames, at least in the sense that there is a class of frames in which the dynamics take a simple and universal form. Knox would say that non-commutative geometry itself seems to select the class of inertial frames and thus plays the role of the structure of space-time.

## 7. Some concluding remarks

Although inertial frame functionalism remains agnostic as to exactly *how* space-time emerges in quantum gravity, in the years following the introduction of this framework into the philosophical analysis of space-time, projects with the direct intent of explaining such emergence in functionalist terms have become central to the field.

In particular, unhappy in their response to the problem of empirical incoherence in Huggett and Wüthrich (2013) and Lam and Esfeld (2013), Lam and Wüthrich (2018, 2020) have attempted to explain spatio-temporal emergence, independently of but inspired by Knox, by recovering those features of relativistic space-time functionally relevant in the production of empirical evidence (with encouraging results in, e.g. string theory, causal set theory and quantum loop theory), condensing their approach in the motto *space-time is as space-time does*.

It seems evident that, whatever the fate of inertial frame functionalism, given its strengths and shortcomings, the idea of space-time as a functional entity will be with us for a long time. Which perhaps should not surprise us, after all, as Daniel Dennett points out, «functionalism (...) is so ubiquitous in science that it is tantamount to a reigning presumption of all of science» (2001, 39).

## Notes

1. Many thanks to Professor Lorenzo Boccafogli, for his comments and continued support without which this article would not have been possible.
2. For the history and philosophy of the concept of space and space-time see Sklar, 1977; Friedman, 1983; Hughett, 2002; DiSalle, 2008 and Maudlin, 2012. For the contemporary substantive-relationalist debate see Earman, 1989 and Pooley, 2013.
3. For the history and philosophy of quantum gravity see Callender and Huggett, 2001; Rickles, 2006, 2020; Huggett, Matsubara and Wüthrich, 2020.
4. For classic discussions of the subject see Stein, 1967, 1977; Earman, 1977, 1979, 1986, 1989; Malament, 1995; Norton, 1995 and DiSalle, 2008.
5. For a critical reading of the dynamic and geometric approach see Norton, 2008, Sus, 2020 and Weatherall, 2020.

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**Kevin Picado Lara** (kevin.picadolara@ucr.ac.cr.com) Philosophy licentiate student at the University of Costa Rica.

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