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ESTIMATION OF THE RELATION BETWEEN WEIBULL DISTRIBUTED SEA CLUTTER AND THE CA-CFAR SCALE FACTOR

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Abstract

When radars operate in coastal or offshore environments, an undesired signal known as clutter appears as background in the measurements. The CA-CFAR detector is the classic solution for the detection of targets inside clutter, typically using a fixed value for its adjustment factor the entire period of operation. Using MATLAB, the author simulates the CA-CFAR response under amplitude samples from the classical sea clutter Weibull distribution. As a result, a relation between operating conditions (Weibull shape parameter) and the most efficient adjustment factor is obtained for various false alarm probabilities. Thus, the implementation of a new detector that achieves the adaptation to changing environments through the use of a variable adjustment factor is suggested. Note that sea environment is present in most radar scenarios both in military and meteorological applications.

Keywords: sea clutter, Weibull distribution, CA-CFAR, false alarm probability, radar targets detection.

Resumen

Cuando un radar opera en ambientes costeros o en alta mar, aparece una señal indeseable denominada clutter marino como fondo en las mediciones. El esquema CA-CFAR es la solución clásica a la detección de blancos sobre fondos de clutter, contando con un factor de ajuste que se mantiene fijo durante todo el período de operación. Empleando la herramienta MATLAB, el autor simula mediciones de amplitud del clásico clutter Weibull bajo la operación CA-CFAR. Como resultado, se obtiene una relación entre las condiciones de operación (parámetro de forma Weibull) y la configuración más eficiente del factor de ajuste para varias probabilidades de falsa alarma. Así, es sugerida la implementación de un detector con factor variable, adaptado a los cambios del ambiente. Nótese que el entorno marino está presente en la mayor parte de los escenarios tanto en aplicaciones militares como meteorológicas.

Palabras clave: clutter marino, distribución Weibull, CA-CFAR, Probabilidad de Falsa Alarma, Detección de Blancos de Radar.

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1. INTRODUCTION

The task of primary radars is to detect objects within the observation area and estimate their position (Barton and Leonov, 1998). Target detection would be an easy task if objects that produce echoes were located on a non-reflecting background. In that case, the echo signal could simply be compared with a fixed threshold, and targets would be detected when the received signal exceeded the threshold (Kouemou, 2009).

However, in real life radar applications, targets almost always appear embedded in a background filled with clutter, which is a random signal. Frequently, the clutter signal's behavior is subject to time and position variations. Therefore, the application of adaptive processing techniques becomes necessary to calculate constantly changing detection thresholds that correspond with the clutter's local situation (Skolnik, 2008). The techniques are even more necessary on widely variable backgrounds such as sea clutter, which is the signal obtained from the radar's echo reflected at the sea surface (Machado and Bueno, 2012).

In order to obtain the necessary local information, schemes with sliding windows around the analyzed cell are commonly used (Rohling, 1983; Nagle, 1991). According on the application, the number of cells to be used in the window may vary, being the larger amounts responsible for a better estimate of the clutter average and the smaller ones more effective at eliminating critical situations, such as: the presence of multiple nearby targets and the occurrence of abrupt changes in the background's level. When such situations occur, the clutter is said to be heterogeneous. Otherwise, it's categorized as homogeneous (Bacallao, 2003).

When detectors are designed for situations where targets appear inside sea clutter, the well-known Neyman-Pearson theorem is applied. This means that the designer first seeks to ensure a given false alarm probability (Pf) and then tries to maximize the probability of detection (Pd). Thus, the most popular clutter level estimation mechanisms are known as CFAR (Constant False Alarm Rate) because they ensure that the detection will occur under the guarantee of a constant false alarm (Skolnik, 2008).

Conceived at first under the assumption of Gaussian distributed clutter, several types of CFAR algorithms can be found, all based on the sliding window mechanism. The most popular are the CA-CFAR (Cell Averaging), the GO-CFAR (Greatest-Of), the SO-CFAR (Smallest-Of) and OS-CFAR (Ordered Statistics). These detectors have been treated in the literature by several authors (Rohling, 1983; Farina, Studer, 1986; Weingberg, 2004). and are often used as a reference on recent researches (Takahashi, 2010; Caso, De Nardis, 2013; de Figueiredo, 2013; Qin, Gong, 2013). . In addition, each year new alternatives and contributions appear in various international journals. Some proposals try to introduce new processing methods (Van Cao, 2012; Qin, Gong, 2013), while others focus on improving the existing ones (Kumar, Kant, 2013; Magaz, Belouchrani, 2011). However, all CFAR

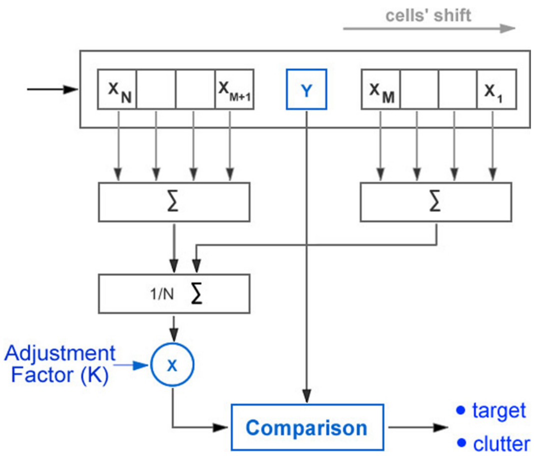

Figure 1. Block Diagram of a CA-CFAR Detector.

implementations have in common that they allow the adjustment of the false alarm probability by means of the modification of a scale or adjustment factor (K), which has an inverse relationship with the probability of detection (Rohling, 1983; Farina, Studer, 1986).

The preliminary statement that claimed the clutter was Gaussian distributed was quickly proven as false by several (Haykin, Bakker, Currie, 2002; Antipov, 1998). Specifically in the case of sea clutter, numerous studies have shown that the family of heavy-tailed distributions is the best suited for representing measurements made on the sea surface. While many others have been proposed, the following distributions are generally the most accepted by the community: Rayleigh, Log-Normal, K, Weibull and Log-Weibull (Antipov, 1998; Oyedokun, 2012; Totir, Rador, Anton, 2008; Jian-bo Hu, Wen-wen, 2009).

1.1 Motivation and objetives

Recent studies have reinforced the theory that sustains that the Weibull distribution is one of the best sea clutter models (Ishii, Sayama, Mizutani, 2011; Sayama, Ishii, 2013). Likewise, it has been noted that the average wave height influences the selection of the shape parameter of the distribution. Additionally, the shape parameter varies when using S-band radars instead of X-band. For S-band radars, the β shape parameter settles around 4,5 and for X-band around 2,5. Besides, if the influence of other not fully specified climatic factors is taken into account, a variation interval which goes from 1,75 to 6,25 may be assumed for the β parameter. The formula for the Weibull Probability Density Function (PDF) is given below (O'Connor, 2011). and draws for several combinations of Weibull parameters are shown in Figure 2.

$$f\,(x|\alpha,\beta) = (\beta x^{\beta-1} / \alpha^{\beta}) \;\; exp\;[-(x/a)^{\beta}] \qquad (1)$$

The previous statements raised questions about whether the selection of a scale invariant factor truly allowed maintaining a constant Pf for the entire operation period of a CFAR detector. Taking as a priori information that the β parameter varies in a known range (Ishii, Sayama, Mizutani, 2011), the ISPJAE radar research group has proved, by performing a number of experiments in MATLAB, that a detector that uses a fixed scale factor must operate inefficiently in order to ensure a constant Pf. On the contrary, if the scale factor would vary according to the value of the β shape parameter from the Weibull sea distribution, the inefficiency will disappear (Machado y Bacallao, 2014).

Thus, the need for a system capable of identifying the β Weibull parameter becomes evident. A project for its creation has already been conceived (Machado, 2014). However, there is an essential add-on that has not been considered as a part of the research. Once identified the Weibull parameter from clutter samples, the systems requires having knowledge about its relation with the CA-CFAR K scale factor, in order to control the variation of the detector's behavior. Therefore, the conceived scheme is useless if a table, with the possible occurrences of β and its corresponding K values, is not available for several false alarm probabilities. The creation of such a table is the objective of the author of this paper.

The CA-CFAR scheme is preferred for the analysis over the SO-CFAR, GO-CFAR and OS-CFAR alternatives because it's the internationally recognized reference model for comparing new implementations. A great amount of recent articles support this statement (Caso, De Nardis, 2013; Qin, Gong, 2013; Kumar, 2013; Ranjan, Krishna Moorthy, 2013; Mashade, 2013).

The current project is intended to solve problems founded in previous researches (Machado, Bueno, 2012; Machado, García, 2014; Machado, García Delgado, B., 2014). where sea clutter and CFAR detectors were studied independently but both approaches were not associated. Neither was conducted a thorough analysis on the behavior of echoes received from sea surface.

Clearly establishing the purpose of the investigation, it should be noted that critical points can't be determined for all false alarm probabilities. This would consume a huge amount of time and efforts. Instead, the author searches to find the combination of values of the CA-CFAR K and the β parameter for which the Pfs are equal

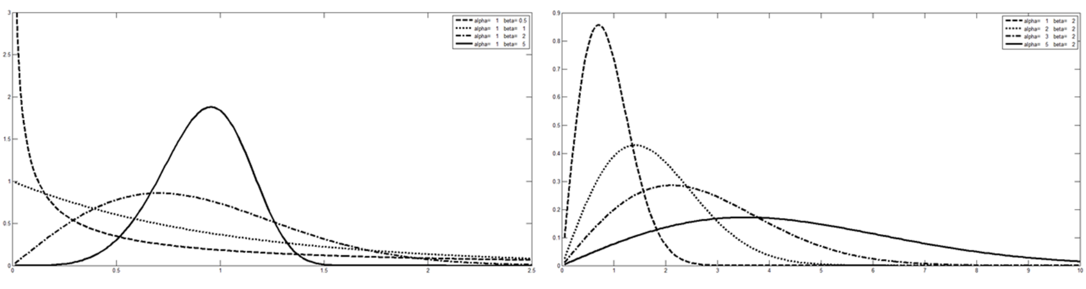

Figure 2. PDF draws from the Weibull Distribution.

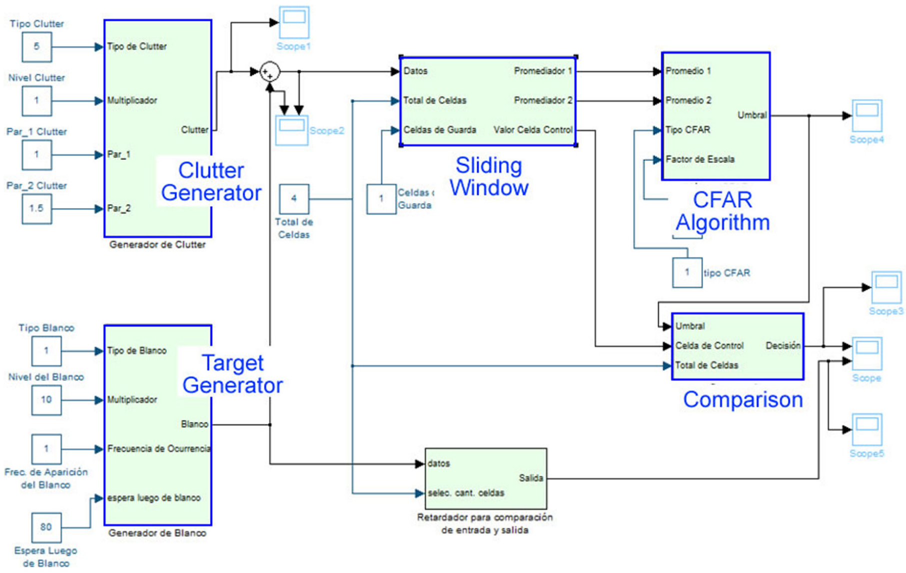

Figure 3. Structure of the MATE-CFAR Test Environment

Table 1. α and β Values Selected for the Simulation.

Beta	**1,75**	**2**	**2,25**	**2,5**	**2,75**	**3**
Alfa	1,1228	1,1284	1,129	1,127	1,1237	1,1198
Beta	**3,25**	**3,5**	**3,75**	**4**	**4,25**	**4,5**
Alfa	1,1156	1,1114	1,1072	1,1033	1,0994	1,0958
4,75	**5**	**5,25**	**5,5**	**5,75**	**6**	**6,25**
1,0924	1,0891	1,086	1,0831	1,0804	1,0779	1,0755

to and. The selection of such Figures is a classic choice in radar radars issues (Kouemou, 2009; Skolnik, 2008).

2. MATERIALS AND METHODS

To achieve the aimed objective, the author worked with the MATE-CFAR (MAtlab Test Environment for CFAR detectors) testing environment created in MATLAB/Simulink 2011 by himself. The software allows the simulation of clutter, targets and CFAR detectors by adjusting the simulation variables in a quickly and intuitively way (Machado, Bacallao, 2014). The multiple blocks that compose MATE-CFAR are shown in Figure 3.

The test environment has a total of 12 configurable parameters that were arranged, in the current research, in a way that Weibull clutter was generated, with no targets present. Besides, samples were processed by a 64 cells CA-CFAR architecture with no guard cells. The previous configuration was maintained all simulation long.

In contrast, the value of the K adjustment factor and the α and β Weibull clutter parameters

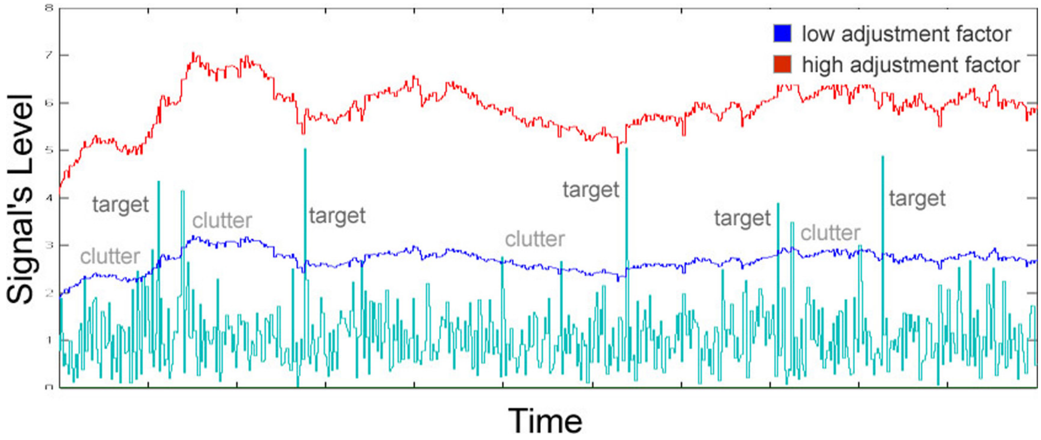

Figure 4. Two possible thresholds obtained by selecting different K values.

were changed until the False Alarm Probabilities of 10^{-2}, 10^{-3} and 10^{-4} were found. The procedure was performed as follows. Firstly, 19 values of β were chosen from the range between 1,75 and 6,25 including the edges, so there was a difference of 0,25 units between consecutives values. The α Weibull parameter was selected for making the average of the samples equal to one, by using the known Weibull mean formula (O'Connor, 2011). given bellow in Equation 2. In addition, Table 1 shows the and values selected for the simulation.

$$mean = \alpha\Gamma\,(1+1/\beta) \qquad (2)$$

Then, using Table 1 as a reference, the first (α, β) pair was placed in MATE-CFAR, and the K was set to a small value such as K = 1,5. Afterwards, one million Weibull clutter samples were processed under the established conditions emulating a real detector. After finishing, the author calculated the Pf according to the following equation:

$$Pf = false\ positives\ /\ total\ amount\ of\ samples \quad (3)$$

Where false positives are those clutter samples mistakenly identified by the CA-CFAR detector as targets. Generally, if the simulation starts by assuming a small K value, the obtained *Pf* is high. Then, as K is increased, in later executions, the Pf will decrease.

The described behavior can be understood if Figure 4 is examined. The reader will notice there are two thresholds resulting from the selection of two different K figures. The lower threshold is associated with a high *Pf* because it often forces to identify clutter samples as targets. Conversely, the higher threshold, corresponding to a higher K, provides a smaller Pf since it's rare to find a clutter value exceeding the established level. However, when a too high threshold is chosen, the detector starts to miss targets that actually exist; so the excessive elevation of K is not recommended because it decreases the Probability of Detection. Consequently, the experiment's goal was to find the exact value for which the scale factor (K) guaranteed a *Pf* of 10^{-2}, 10^{-3} and 10^{-4} for each pair of Weibull parameters with the best possible value of probability of detection.

As the reader may deduce, after obtaining the *Pf* for the first K value, the procedure was repeated changing the K value until the desired Pf was found. About 20 executions were necessary as an average to ensure a good accuracy on the results extracted from the experiment (less than 0,5 % of error).

After finishing with one (α, β) pair, the next one was be included in the trials, performing simulations until all the required Ks were founded. The procedure was completed when all pairs were processed. A summary of the algorithm is described below.

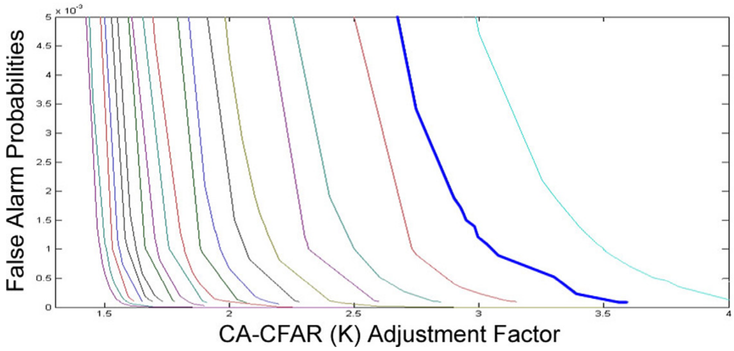

Figure 5. Relation between the K Adjustment Parameter and the False Alarm Probability for several.

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Repeat 19 times
In each Repetition: Select a (α, β) pair
Repeat until the Pf is found (approximately 20 repetitions)
      In each repetition: Modify K looking for the desired Pf
End of Repetitions
End of Repetitions
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3. RESULTS

After performing the procedure described in the preceding section, the data shown in Figure 5 was obtained. The X axis provides the CA-CFAR K value and the Y axis the calculated Pf. Each draw corresponds to one β Weibull shape parameter. Thus, 19 draws are visible covering the range from 1,75 to 6,25. The draw to the extreme right shows the result for the smaller β (β = 1,75). Draws appearing to the left plot lines related with the gradual increase of the shape parameter.

As it may be observed in Figure 5, the slope was always negative in what constitutes an expected result if the effect of raising the adjustment factor is taken into account. Note that by increasing K, the calculated threshold gets higher, so fewer targets will be classified as clutter obtaining as a consequence a smaller false alarm probability.

Figure 6 shows one of the draws from Figure 5 (the one highlighted) a little more in detail. In this case, besides the solid line, some dots in a cross shape were plotted. Each dot represents a measurement of *Pf* obtained by performing an experiment with one million samples using the procedure described in the previous section.

As it can be seen, most of the crosses are distributed in three concentration areas. Indeed, the groups of measurements are around the *Pf* values of 10^{-2}, 10^{-3} and 10^{-4}. The reader may understand then that the search for K was not made in a uniform manner but it was performed efficiently by getting closer and closer to the points of interest. However, outliers appeared as a result of the lack of information of the initial executions.

The result of the measurements was the extraction of the desired critical point where the required Pf was obtained. Table 2 relates each occurrence of the β Weibull parameter with the better CA-CFAR K. The second column displays the values of the shape parameter used in the simulation, while the rest to the right show the values of the best possible K for each tabulated *Pf*. Note that each of the amounts shown was extracted from Figure 5.

In order to maximize the practical application of the values shown in Table 2, the author recommends focusing future research efforts in the development of a mathematical expression that will allow the generalization of the results. With this expression, it will be possible to estimate the K value necessary to maintain the desired *Pf* for any β in the range from 1,75 to 6,25, and not only for the values tabulated in Table 2.

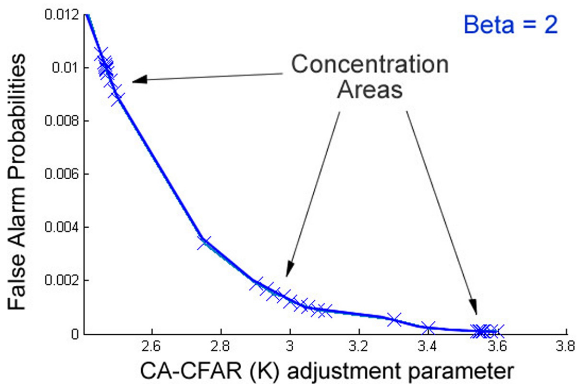

Figure 6. Successive Executions for the precise measuring of the adjustment parameter.

Table 2. Best k Scale Factor for Several Clutter Configurations.

N,	β Weibull Shape Par	K for $P_f = 10^{-2}$	K for $P_f = 10^{-3}$	K for $P_f = 4$
1	1,75	2,74	3,50	4,16
2	2,00	2,47	3,05	3,55
3	2,25	2,26	2,73	3,13
4	2,50	2,11	2,50	2,82
5	2,75	1,99	2,32	2,82
6	3,00	1,89	2,18	2,41
7	3,25	1,81	2,06	2,27
8	3,50	1,74	1,96	2,15
9	3,75	1,68	1,88	1,99
10	4,00	1,63	1,82	1,99

4. DISCUSSION

The founded relation between the K and the Weibull shape parameter achieved an important step toward the creation of an improved CA-CFAR detector, which will vary its adjustment factor according to the conditions of the environment. The new detector would really guarantee that a constant *Pf* is maintained when facing variable operating conditions. However, the conception of the scheme contains other complexities not addressed in the current research such as the number of samples to be taken from real clutter measurement for guarantying stability in the β Weibull parameter.

Making a preliminary analysis of the observed data, it was convenient to plot the difference

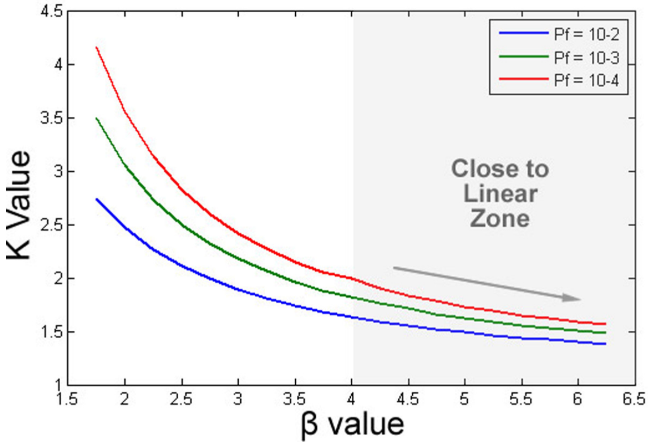

Figure 7. Derivative of the K founded values.

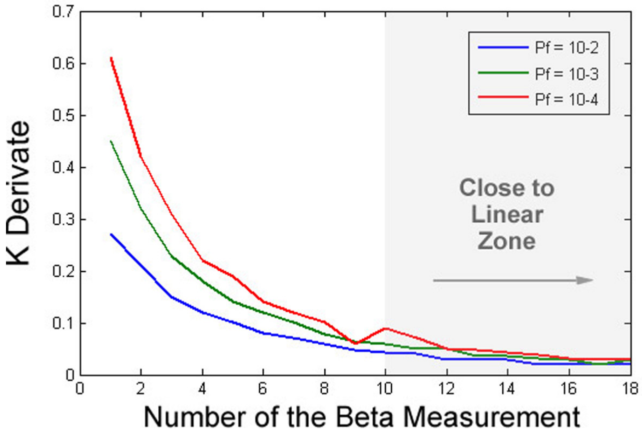

Figure 8. β influence on K for several False Alarm Probabilities.

between each of the values of table II, that is, the subtraction of each value with the subsequent. So, Figure 7 shows the derivative of the K value for each of the addressed Pfs. Note that this time the X axis shows the 18 values resulting from the subtraction of the 19 K measurements.

As it can be seen, plots were decreasing and followed a non-linear structure. This indicates that it's highly unlikely that small order polynomial approximations will fit the data correctly. However, the derivative seemed to stabilize after the 10th measurement which corresponds to $\beta = 4$ (see Table 2). Consequently, an estimator of the measurements could be divided into two parts if good results are not reached by using a single one.

The area with greater linearity is also visible in Figure 8 where data is shown before performing the derivation. Note that this Figure illustrates the influence of β on K.

From Figure 7 and 8, it was visible that as β increases, its effect on the modification of K becomes reduced. The fact is remarkable when trying to establish expressions to generalize the obtained data. Selected expressions must have a smooth behavior beyond the $\beta = 6{,}25$ limit, as it was suggested by the shapes of the draws. Besides, the expressions must have a tendency to converge for the higher βs and to divergence for smaller βs.

On another note, observe that the shapes of the draws in Figure 8 were very similar. This suggests that a single expression could reproduce the measurements of the three Pfs. An approach which could prove to be satisfactory is to select a base function which will be added to an auxiliary function that will change according to the selected Pf. The author did not tried to find expression to generalize the results because he thought that higher amounts of samples should be used to calculate better false alarm probabilities before offering mathematical expressions.

5. CONCLUSIONS

The best values for the K scale factor where found for a CA-CFAR detector and false alarm probabilities of 10^{-2}, 10^{-3} and 10^{-4}, considering several states of Weibull distributed clutter. The founded figures guarantee that a CA-CFAR scheme will operate maintaining its theoretic false alarm probability while the probability of detection is maximized, even when facing variations of the β clutter parameter from 1,75 to 6,25. In addition, the study showed that for the upper values of β, the clutter influence decreases over the correction of the K factor. The current investigation concluded an important piece of a system designed to operate as an improved CA-CFAR detector that will adapt to variations in the environment.

6. RECOMMENDATIONS

The author recommends finding a mathematical expression through which the founded results may be reproduced. Likewise, it will be necessary to increase the amount of samples involved in the experiment in order to calculate higher order false alarm probabilities.

The reproduction of the study for the classic OS-CFAR detector is also recommended. This would begin creating a general methodology for the design of detectors adapted to clutter conditions.

It will be also necessary, for the further development of the proposed scheme, to estimate the convergence of the Weibull clutter to its statistical mean as more samples are taken into consideration. This will allow establishing the necessary quantity of samples to gather for performing the proposed K correction. Note that the amount of samples should be inferior to one million.

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