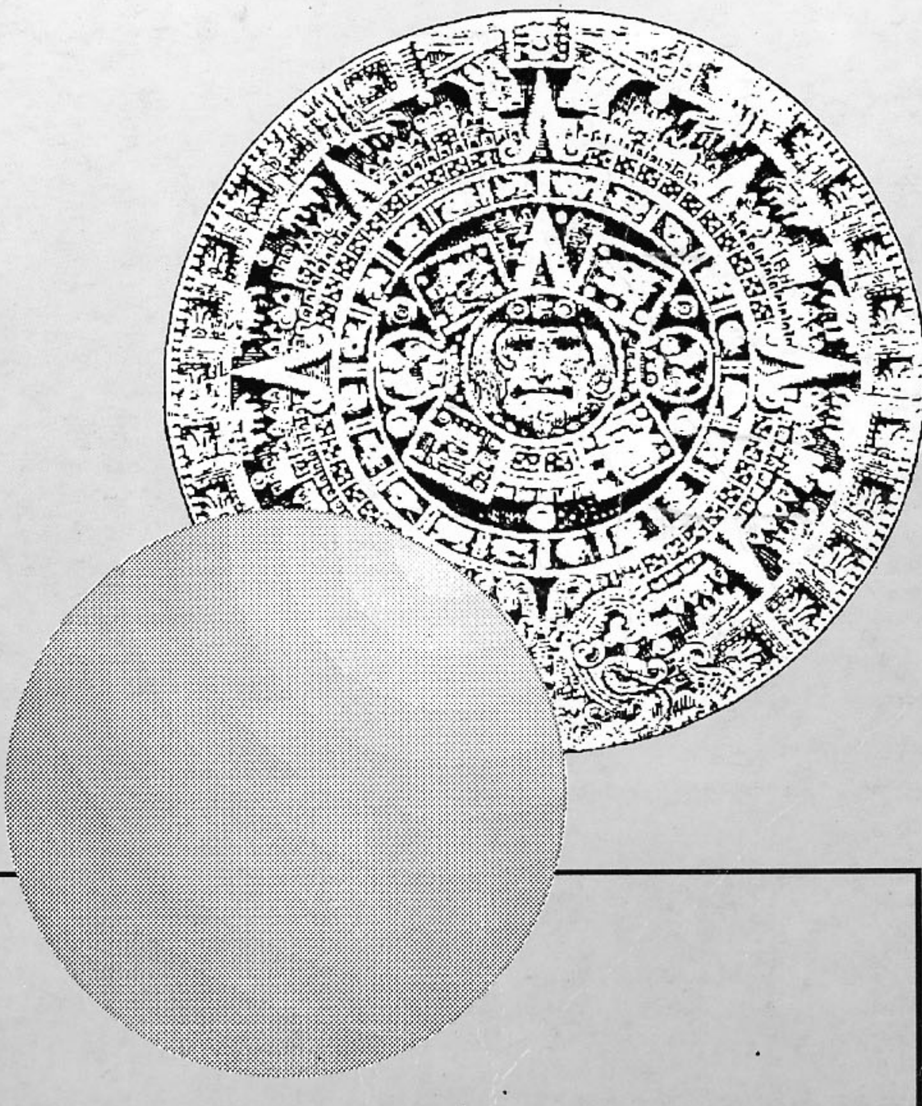
# Ingeniería

Revista de la Universidad de Costa Rica
ENERO/JUNIO 1992 VOLUMEN 2 No. 1





INGENIERIA

1992

# INGENIERIA

Revista Semestral de la Universidad de Costa Rica

Volumen 2 Enero-junio 1992 Número 1



**CORRESPONDENCIA Y SUSCRIPCIONES**

Editorial de la Universidad de Costa Rica
Apartado Postal 75
2060 Ciudad Universitaria Rodrigo Facio
San José, Costa Rica

**CANJES**

Universidad de Costa Rica
Sistemas de Bibliotecas, Documentación e información
Unidad de Selección y Adquisiciones-CANJE
Ciudad Universitaria Rodrigo Facio
San José, Costa Rica

**Suscripción anual:**
Costa Rica:¢500,00
Otros países: US $20.00

**Número suelto:**
Costa Rica:¢250,00
Otros países: US $10.00

Ingeniería 2 (1): 43-50, 1992 San José Costa Rica

# STEADY-STATE ANALYSIS OF THE PARALLEL RESONANT CONVERTER

*Emilio Alpizar Villegas**

**Abstract:**

**In this work a survey of methods for the steady state analysis of the parallel resonant converter is made for its operation above resonance.**

**Sumario:**

**En este trabajo se present a un resumen de métodos para el análisis, en estado estable, del convertidor resonante con conexión en paralelo. El análisis es válido para operación a frecuencias arriba de la frecuencia de resonancia.**

## INTRODUCTION

In order to reduce the size of the power supplies intended to be used in modern computing systems, it is desirable to raise their operation frequency because it will reduce the size of the magnetic components. But this increasing in operating frequency also will increase the amount of power losses by mean of the switching operation. High-frequency resonant converters, whether parallel or series, have become increasingly popular among power supply designers, specially at frequencies above 100 Khz, because they offer small size, good reliability, and reduced EMI/RFI. Recent advances and price reductions in both control Ics and power MOSFETs have made resonant sine wave converters even more popular.

Some of the advantages of the resonant sine wave converter,compared to other conventional topologies, follow: With this converter, there is a higher overall efficiency at a given power level, mainly due to the absence of switching losses at the power switch and the rectifiers. Lower losses in turn mean smaller heat sinks, hence reduction in size and weight of the overall package. Because the voltage is switched when the drain current of the MOSFET switching transistors is zero, operation at higher frequencies is possible, resulting in smaller magnetic elements and filter components.

## RESONANT TOPOLOGIES

There are three basic types of resonant converter topologies as shown in Figure 1. The terms series, parallel, and series-parallel refer to the connection mode of the load to the converter, not to the connection of the resonant components.

In the series loaded circuit, the two capacitors Cs/2 form a series resonant capacitor of value Cs. For the parallel loaded converter Cp is the only resonant capacitor, while the capacitor Cin/2 serves only to split the input dc voltage. The series parallel has both series and parallel resonant capacitors.

Those circuits may be operated either above or below resonant frequency.

*Universidad de Costa Rica Escuela de Ingeniería Eléctrica

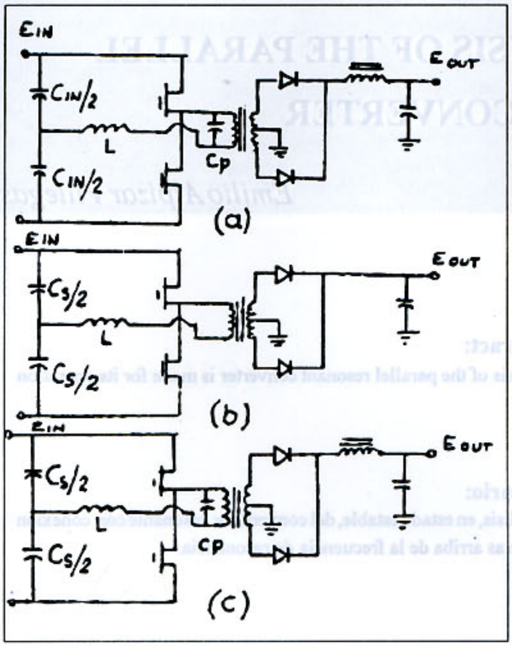


**Figure 1.**
Three types of half-bridge resonant converts. a) Parallel. b) Series, c) Series Parallel.

Figure 2. shows a typical topology for the parallel resonant converter where:

| | |
|---|---|
| S1,S2 | : High frequency switches |
| D1,D2 | : Antiparallel diodes across S1 and S2. |
| L,C | : Only inductor and capacitor resonant components |
| HFT | : High frequency transformer |
| SNUBBER | : RC for below and C for above resonance |
| Ld | : Smoothing output inductor |
| RL | : Load resistance |
| D3,D4,D5,D6 | : Diodes of the ouput rectifier bridge. |

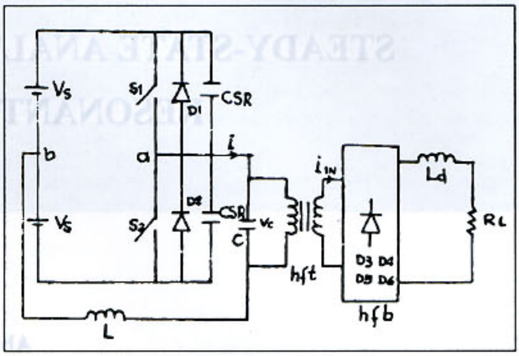


**Figure 2.**
A dc/dc high frecuency link parallel resonant converter for operation above resonance

## OPERATION OF THE PRC

In a resonant converter, the half bridge applies a square wave of voltage to the resonant circuit and, due to the filtering action of the resonant circuit, approximate sine waves of current are presented in the resonant inductor. Typical waveforms for the PRC of Figure 2 are shown in Figure 3. In Figure 3. the following conditions are given:

D1 is closed from 0 to T1, S1 is closed from 0 to T2, D2 is closed from T2 to T3, T2 is closed from T3 to T4, and,

$V_c$: Voltage across the capacitor,
$V_s$: Peak voltage of the source
$V_{co}$: Initial value of $V_c$ at time D1 closes
$I_o$: Initial value of IL at time D1 closes

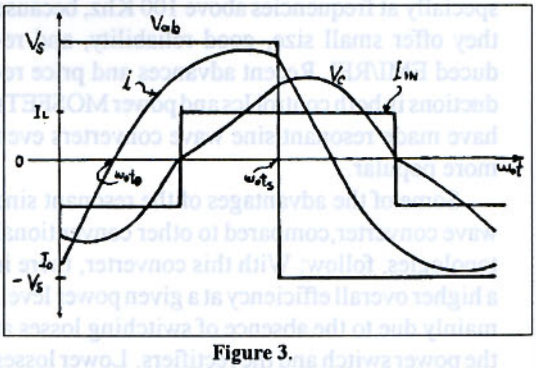


**Figure 3.**
Wave form, fo r operation above resonance, of the PRC on Figure 2

To explain the waveforms of figure 3. it is assumed that diode D1 is conducting initially. When the current through D1 reaches zero, switch S1 is turned on and the current is transferred to it from the adjacent diode D1. A resonating current flows through the switch into the supply via the resonating components L and C. There is no voltage across the switch at turn on ( since D1 is conducting), which eliminates the turn-on losses and facilitates the operation of the switches. When switch S1 is turned off, current is transferred to D2 from S1. Turn-off losses are present in such type of operation because voltage and current are present at that time. The sequence of events repeats in the next half of the cycle for diode D2 and switch S2.

The load current can be considered constant because of the large inductance Ld at the ouput of the bridge rectifier and an ouput voltage ripple of the order of twice the triggering frequency.

The current direction at the input rectifier changes according to the polarity of capacitor voltage Vc. Depending on the polarity of the capacitor voltage, the diodes conducting in the rectifier bridge are different ones. This causes the inverter to enter into different modes of operation. In one mode, the capacitor voltage Vc is of the same polarity as that of Vab while in the other, it is the opposite polarity. Whenever a switch is turned off , current gets transferred from it to the diode in the other half of the bridge and the following mode commences. This mode ends when the polarity of the capacitor voltage changes and causes a different set of ouput rectifier diodes to conduct. The occurrence of modes is sequential, and what is considered a mode depends on the author point of view and the technique used to analyze it.

## ANALYSIS, THE GENERAL SOLUTION

Complicated and seemingly unsolvable algebraic equations result when analyzing the parallel resonant converter. This apparent difficulty can be avoided and the analysis made simpler without loosing accuracy using the following assumptions:

1. The switches and diodes used in the converter are z ideal.
2. The resonating components L and C are lossless.
3. The high frequency transformer and the input D.C source are ideal.
4. Effect of snubber capacitors is neglected.
5. The filter inductance is large enough to assume constant current model.
6. The transformer and the source are ideal.

The converter operates in several modes, depending on the state of the diodes of the half bridge but these states also depend on the polarity of the capacitor (sign of Vc). In one mode, the capacitor voltage Vc is of the same polarity as that of Vab, while in the other, it is the opposite polarity. The operation of the PRC can be evaluated from one equivalent circuit common to the different modes of operation. This circuit is depicted in Figure 4, where E denotes equivalent. Choosing the voltage across the capacitor and the current through the inductance as the two state variables, we have:

$$I_L = I_C + I_E = C\frac{dV}{dt} + I_E \tag{1}$$

$$\frac{dV_C}{d_t} = \frac{1}{C}I_L - \frac{1}{C}I_E \tag{2}$$

$$V_c = V_E - V_L = V_E - L\frac{dI_L}{d_t} \tag{3}$$

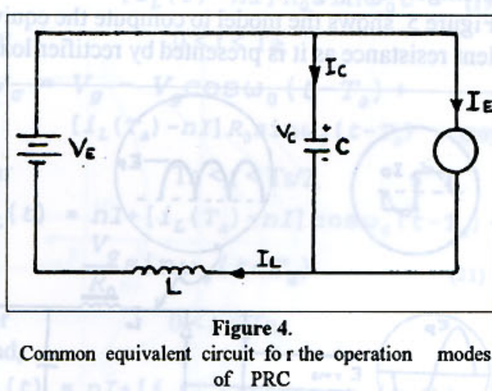



**Figure 4.**
Common equivalent circuit fo r the operation modes of PRC

$$\frac{dI_L}{dt} = \frac{1}{L}V_E - \frac{1}{L}V_c \tag{4}$$

Then, the system is of the form:

$$\begin{bmatrix} \frac{d_i}{d_t} \\ \frac{dV_c}{d_t} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} i \\ Vc \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & -\frac{1}{C} \end{bmatrix} \begin{bmatrix} E \\ I_L \end{bmatrix} \quad (5)$$

The state variable representation in matrix form is:

$$[\dot{x}] = [A]\,x + [B]\,u \quad (6)$$

Where u will be different depending on the sign of Vc, that is:

1) u = [+Vs +IL]$^T$
2) u = [-Vs +IL]$^T$
3) u = [+Vs -IL]$^T$
4) u = [-Vs -IL]$^T$

Here "u" will determine the topology mode for each case.

## ANALYSIS, STEADY STATE SOLUTION

The solution to this system set of equations can be done by several ways, including the exact analysis and methods of approximate solution. Some of these methods are presented here.

1. Classical complex analysis:

This method is given in the paper of Steingerwald [2].

Figure 5. shows the model to compute the equivalent resistance as it is presented by rectifier loads.

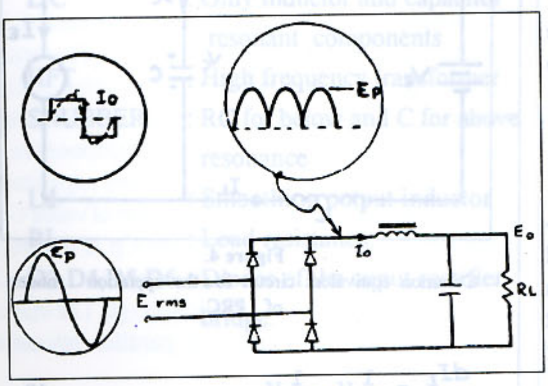



**Figure 5.**
Voltage source drive model for a PRC

From the figure, we have

$$R_L = \frac{E_0}{I_0} \quad (7)$$

$$E_{ac}(rms) = \frac{\pi}{2\sqrt{2}} E_0 \quad (8)$$

$$I_{ac}(rms) = 2\frac{\sqrt{2}}{\pi} I_0 \quad (9)$$

$$E_0 = \frac{2}{\pi} E_p = \frac{2}{\pi}\sqrt{2} E_{ac}(rms) \quad (10)$$

$$R_{ac} = \frac{E_{ac}(rms)}{I_{ac}(rms)} = \frac{\pi^2}{8} R_L \quad (11)$$

The parallel resonant converter is then modeled, using the equivalent resistance as in Figure 6.

From Figure 6, the ac gain for the circuit is:

$$\frac{V_o}{V_I} = \frac{1}{1 - \frac{X_L}{X_c} + j\frac{X_L}{R_{ac}}} \quad (12)$$

In this case

$$V_o = \frac{\pi}{2\sqrt{2}} E_0 \quad (13)$$

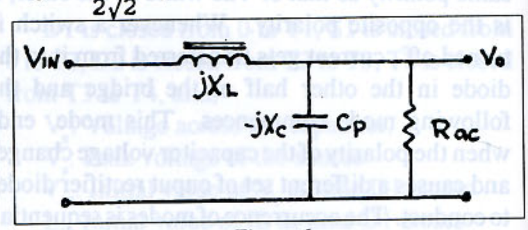



**Figure 6.**
Equivalent circuit for the PRC

and

$$V_I = 2\frac{\sqrt{2}}{\pi} E_d \quad (14)$$

Where

$$E_d = \frac{V_I}{2} \quad (15)$$

Defining

$$Q = \frac{R_L}{\omega_0} \quad (16)$$

The gain of the parallel resonant converter is finally given as:

$$\frac{E_0}{E_d} = \frac{1}{\frac{\pi^2}{8}\left[1-\left(\frac{\omega}{\omega_o}\right)^2\right] + j\frac{\omega}{\omega_0}\left(\frac{1}{Q}\right)} \quad (17)$$

This equation is plotted in figure 7.

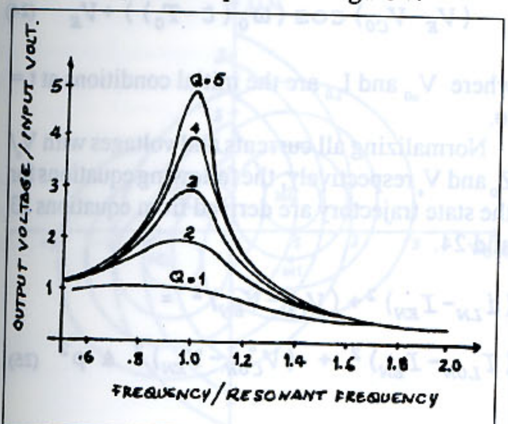



**Figure 7.**
Parallel resonant converter gain

It is noted that the maximum output occurs near resonance for Q's above 2 and that the maximum output voltage can be computed from

$$\left[\frac{E_0}{E_I}\right]_{max} = Q \quad (18)$$

Those curves are accurate above resonance, where the resonant circuit filters the harmonics present on the input square wave.

From Figure 7. it is noted that the converter is able to control the output voltage at no load by running at a frequency above resonance. Also it is seen that the output voltage at resonance is a function of load and it may rise to very high values at no load if the operating frequency is not raised by the regulator.

2. Closed Form Solution.

The closed form expressions for the tank waveforms are given by Johnson and Erickson[3]. For the continuous conduction mode there are two different topology modes as is shown in figure 8. For the analysis it is considered that the transition between modes occurs when Vc passes through zero and the second pair of diodes in the output bridge starts to conduct.

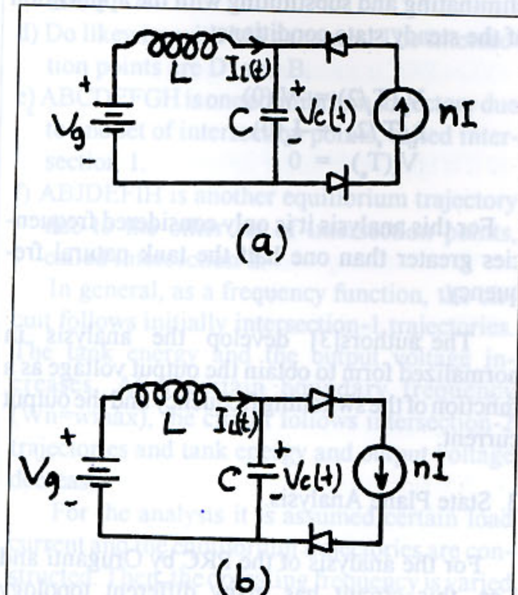



**Figure 8.**
PRC equivalent circuits

The equations for voltage and current on those circuits are written in sinusoidal form as:

$$V_c(t) = V_g + [v_c(0) - V_g]\cos\omega_o t + [i_L(0) + nI]R_0\sin\omega_o t \quad (19)$$

for $0 < t < Ta$,

$$V_c = V_g - V_g\cos\omega_0(t-T_a) + [i_L(T_a) - nI]R_0\sin\omega_0(t-T_a) \quad (20)$$

for $Ta < t < Ts/2$,

$$i_L(t) = nI + [i_L(T_a) - nI]\cos\omega_0(t-T_a) + \frac{V_g}{R_0}\sin\omega_0(t-T_a) \quad (21)$$

for $0 < t < Ta$,
and,

$$i_L(t) = nI + [i_L(T_a) - nI]\cos\omega_0(t-T_a) + \frac{V_g}{R_0}\sin\omega_0(t-T_a) \quad (22)$$

for $Ta < t < Ts/2$ ; $Vc(Ta) = 0$

In previous equations wo=1/LC is the tank natural frequency, and $R_o$=L/C is the tank characteristic impedance. In those equations $T_a$, $I_L(0)$, $V_c(0)$ and $I_L(T_a)$ are unknowns which must be found. To find them it is followed a process of eliminating and substituting with the application of the steady state conditions:

$$V_c(T_a/2) = -V_c(0)$$
$$I_L(T_a/2) = -I_L(0)$$
$$V_c(T_a) = 0$$

For this analysis it is only considered frequencies greater than one half the tank natural frequency.

The authors[3] develop the analysis in normalized form to obtain the output voltage as a function of the switching frequency and the output current.

## 3. State Plane Analysis.

For the analysis of the PRC by Oruganti and Lee, this circuit has many different topology modes, where four of them are resonant and five are non resonant topologies. These many modes give the difficulties in the PRC analysis.

The solution to the network in Figure 4. is given in the following form:

$$i_L = \frac{V_E - V_{C0}}{Z_0}\sin(\omega_0(t-t_0)) + (I_{L0}-I_E)\cos(\omega_0(t-t_0)) + I_E \quad (23)$$

$$V_C = (I_{L0}-I_E)Z_0\sin(\omega_0(t-t_0)) - (V_E-V_{C0})\cos(\omega_0(t-T_0)) + V_E \quad (24)$$

where $V_{c0}$ and $I_{L0}$ are the initial conditions at t = to.

Normalizing all currents and voltages with $V_E/Z_0$ and $V_E$ respectively, the following equations for the state trajectory are derived from equations 23 and 24.

$$(i_{LN}-I_{EN})^2 + (V_{CN}-V_{EN})^2 = (I_{LON}-I_{EN})^2 + (V_{CON}-V_{EN})^2 \triangleq \rho^2 \quad (25)$$

The subscript N refers to normalized circuit variables. Equation 25 is the equation of a circular trajectory. The state trajectories are circles with radius p, which depends on the initial conditions, and with center located at (VEN,IEN).

Based on the equation 25, four sets of state trajectories,one for each topological mode, are shown in figure 9.

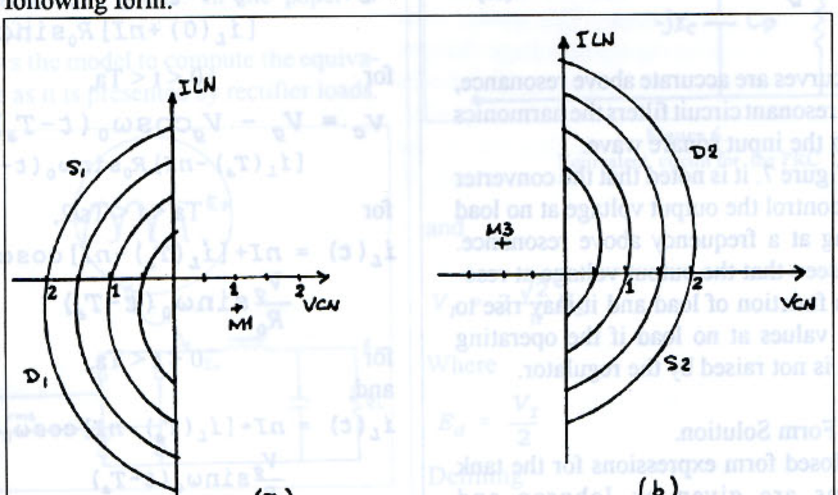



**Figure 9.**
State trajectories for $I_{ON}$=.5.

The state trajectories extend to only half of the VCN-ILN plane because each topological mode corresponds to only one polarity of the voltage across the capacitor. By combining these sets of trajectories, a composite state portrait, which completely characterizes the PRC at a given load current, can be constructed as in Figure 10.

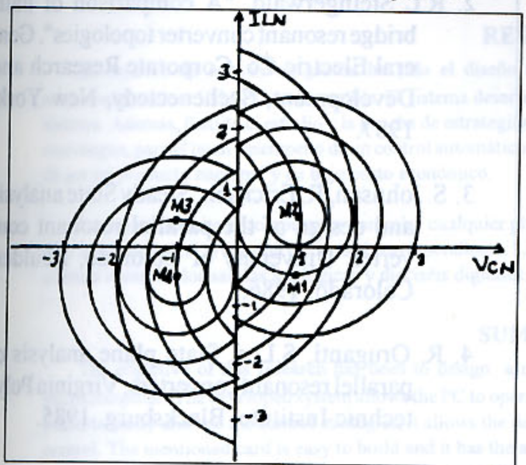



**Figure 10.**
State portrait of the for $I_{ON}$=.5 and center $M^1$,$M^2$,$M^3$, and $M^4$

For given operating frequency and load, a closed equilibrium trajectory curve that corresponds to the steady state waveforms can be drawn on the state-plane. One of those curves is marked in Figure 10. with a sectioned line.

Having the load current, the construction of steady state equilibrium trajectories is illustrated in figure 11. The steps are as follows:

a) Locate the mode centers M1,M2,M3,M4.
b) With centers at M2 and M4, Draw arcs ABCD and EFGH, respectively, with a fixed radius R.
c) With M1 as center, draw arc AHIF of radius R'(=M1A). This arc intersects arc EFGH at H and F.
d) Do likewise with M3 as center. The intersection points are D and B.
e) ABCDEFGH is one equilibrium trajectory due to one set of intersection points, called intersection 1.
f) ABJDEFIH is another equilibrium trajectory due to the other set of intersection points, called intersection 2.

In general, as a frequency function, the circuit follows initially intersection-1 trajectories. The tank energy and the output voltage increases. At a certain boundary frequency (Wn=wmax), the circuit follows intersection-2 trajectories and tank energy and output voltage decrease.

For the analysis it is assumed certain load current and the equilibrium trajectories are constructed. Then, the operating frequency is varied in order to get the different modes occurring in the PRC and boundaries among them. The above process is repeated for different load currents. In this manner, the operation of the PRC is studied for the entire useful range of frequencies and load currents.

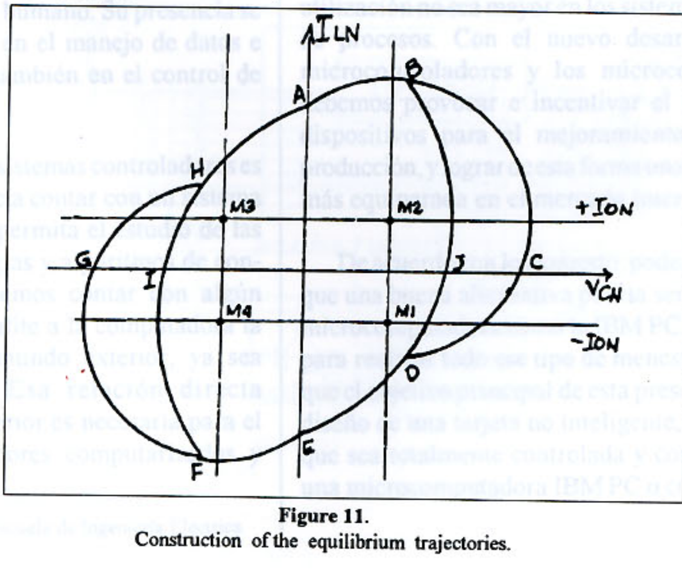



**Figure 11.**
Construction of the equilibrium trajectories.

## CONCLUSION

The analysis of the Parallel Resonant Converter can be done by several methods. Some of those methods have been described here for the steady state operation. Among the many characteristics the PRC has we find relevant the following:

a) The PRC is suitable for low output voltage, high output current applications.

b) It is applicable under severe short circuit conditions.

c) The switching losses are very low.

d) The current is practically independent of load.

e) The voltage can be controlled over a wide range of frequency.

f) It is not suitable for use with loads lower than 75% the design load.

## BIBLIOGRAPHY

1. AKS. Bhat, M.N Swammy. "Analysis and design of a high frequency parallel resonant converter operating above resonance", University of Victoria, Canada, 1988.

2. R.L Steingerwald. "A comparison of half-bridge resonant converter topologies". General Electric Co. Corporate Research and Development, Scchenectedy, New York, 1987.

3. S. Johnson, R. Erickson. Steady State analysis and design of the parallel resonant converter. University of Colorado, Boulder, Colorado, 1986.

4. R. Oruganti, S.Lee. State plane analysis of parallel resonant converters. Virginia Polytechnic Institute, Blacksburg. 1985.

5. G. Chryssis. High-Frequency Switching Power Supplies. Second Edition, McGraw Hill, 1989.