© Revista de Matemática: Teoría y Aplicaciones 2023 **30**(1) : 1–23
cimpa – ucr issn: 1409-2433 (Print), 2215-3373 (Online)
doi: https://doi.org/10.15517/rmta.v30i1.50554

INFORMATION QUANTIFIERS AND UNPREDICTABILITY IN THE COVID-19 TIME-SERIES DATA

CUANTIFICADORES DE INFORMACIÓN E IMPREDICTIBILIDAD EN LAS SERIES TEMPORALES ASOCIADAS A LA COVID-19

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Received: 01/April/2022; Revised: 03/Aug/2022; Accepted: 18/Oct/2022

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Abstract

We apply different information quantifiers to the study of COVID-19 time series. First, we analyze how the fact of smoothing the curves alters the informational content of the series, by applying the permutation and wavelet entropies to the series of daily new cases using a sliding-window method. In addition, to study how coupled the curves associated with daily new cases of infections and deaths are, we compute the wavelet coherence. Our results show how information quantifiers can be used to analyze the unpredictable behavior of this pandemic in the short and medium terms.

Keywords: information theory; permutation entropy; statistical complexity; Bandt-Pompe methodology; wavelet transform.

Resumen

Aplicamos diferentes cuantificadores de información al estudio de series temporales de COVID-19. En primer lugar, analizamos cómo el hecho de suavizar las curvas altera el contenido de información de la serie, aplicando la entropía de permutaciones y la entropía wavelet a la serie de casos diarios nuevos mediante un método de ventana móvil. Además, para estudiar qué tan acopladas están las curvas asociadas con los nuevos casos diarios de infecciones y muertes, calculamos la coherencia wavelet. Nuestros resultados muestran cómo se pueden utilizar cuantificadores de información para analizar el comportamiento impredecible de esta pandemia en el corto y mediano plazo.

Palabras clave: teoría de la información; entropía de permutaciones; complejidad estadística; metodología de Bandt-Pompe; transformada wavelet.

Mathematics Subject Classification: 92-10, 92-11.

1 Introduction

Information quantifiers have a widespread use in many areas of research [12, 29, 10, 1, 30, 5]. In particular, they have been useful to characterize the informational content of time series of complex systems [14, 21, 13, 28]. In this work, we analyze the time series associated with the COVID-19 pandemic by appealing to different information measures, in order to gain a better understanding of its dynamics. For related studies on the subject, see [11, 15, 7, 8].

In general, the different indicators associated with the pandemic (such as the records of daily new cases and daily deaths) show a quite unpredictable behavior. Many works have focused in trying to model their dynamics, obtaining a reasonable degree of accuracy. However, predicting the behavior of the time series remains challenging in several respects and, in particular, it is difficult to understand the fluctuations that occur in the short and medium terms. It is relevant to properly understand these fluctuations, since they are often used as indicators of the evolution of the pandemic in discussions about public health policies. As we show below, the information quantifiers give us an interesting characterization of those fluctuations.

In a previous work, we used the wavelet and permutation entropies to study the COVID-19 data series [11]. Due to the high entropy obtained, we concluded that the series have low informational content. The meaning of the previous assertion is that its behavior is highly unpredictable. Here, we extend the previous study and look more closely to other aspects of the time series.

We analyze how the entropic quantifiers depend on the smoothness of the curve. Short-term fluctuations in the time series are expected to increase entropy. Thus, if a preprocessing of the available data is applied in such a way that it results in a smoothed time series (as for example, by applying a moving average), thus decreasing the fluctuations, we expect the entropy to decrease as well. We show how this phenomenon takes place by appealing to a sliding-window method [10]. The sliding-window analysis is important for our purpose, because it allows to study the evolution in time of the information content of a time series. In this way, it is possible to characterize different phases in the time evolution of a variable. First, we explain how to use the entropic quantifiers to extract maximal information about the medium and long-term behavior of the time series. Next, we notice that, even if the entropy decreases, the mean values obtained are still high. Such a result can be interpreted as follows: the high degree of unpredictability is intrinsic to the dynamics of the pandemic.

Finally, we consider the problem of characterizing the interdependence of different time series using information quantifiers. We do this by analyzing the wavelet coherence between the time series of daily new cases and daily deaths in a given country. As the wavelet coherence gives information in time and frequency by using finite time windows, it is suitable for analyzing nonstationary time series. Our results suggest that wavelet coherence is a relevant tool to quantify changes in the dynamics of the daily new cases time series. In particular, it seems that, in some cases, it could be used for quantitatively assessing the effect of health policies, such as vaccination campaigns. The paper is organized as follows. In Section 2, we briefly review the basic aspects of the information quantifiers that will be used in this work. In Section 3, we describe the sliding-window method to study the variation in time of the permutation and wavelet entropies. The calculations for a test signal are presented in 3.1.

In 3.2 we analyze different countries and we show how their mean values change when the curves are smoothed. In Section 4, we compute for two cases the wavelet coherence between daily new cases and daily deaths curves, and we show how they decouple after the vaccination campaigns. Finally, in Section 5 we draw some conclusions.

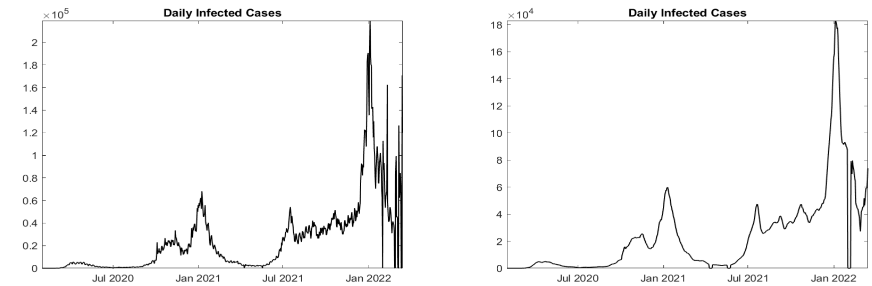

Figure 1: United Kingdom. Daily cases $l = 1$ (left), and smoothed curve using moving average $l = 7$ (right).

2 Entropic quantifiers

In this section, we review some basic aspects of the information quantifiers used in this work. In what follows, we restrict the presentation to the discrete case.

2.1 Permutation entropy

The permutation entropy was introduced in reference [2] (see also references [23, 24, 25] for more details; see [10, 16] for particular examples of applications). Given a time series $\{x_i\}_{i=1}^{T}$, an associated probability distribution is built as follows. First, we choose two parameters: the embedding dimension $D \in \mathbb{N}$ and the time delay $\tau \in \mathbb{N}$. Next, for each time $k = 1, \ldots, T - (D-1)\tau$, we consider the sequence $I_k = (x_k, x_{k+\tau}, \ldots, x_{k+(D-1)\tau})$. To each I_k it is assigned a permutation pattern, i.e. a permutation of the set $\{1, \ldots, D\}$, according to the following criterion:

$$I_k \;\rightarrow\; \pi_l = (m_0, m_1, \ldots, m_{D-1}), \quad 1 \leq l \leq D!\,, \tag{1}$$

with $m_0, m_1, \ldots, m_{D-1}$ such that $x_{k+m_0\tau} < x_{k+m_1\tau} < \ldots < x_{k+m_{D-1}\tau}$ (see [26]). This means that a permutation pattern π_l is assigned to each sequence I_k. This pattern keeps track of the permutations that result from reordering it from lowest to highest. Thus, a collection of permutation patterns is associated with the set $\{I_k\}_{k=1,T-(D-1)\tau}$. From the collection of permutation patterns, we can obtain a normalized histogram which gives place to a probability distribution. The frequency of a given pattern π is given by

$$p(\pi) = \frac{\#\{k \mid k \leq T - D\,,\ I_k \text{ has type } \pi\}}{T - D + 1}. \tag{2}$$

In this way, a probability vector is assigned to the time series, out of which it is possible to define the permutation entropy as

$$S_P(p) = -\frac{1}{\ln(D!)} \sum_{\pi} p(\pi) \ln(p(\pi)), \tag{3}$$

the permutation entropy can be easily implemented in Python or MATLAB codes (here we use a variant of the MATLAB code that can be found in [19]; see also [14, 21]). How equation 2 is applied to obtain the probability distribution is illustrated in the following simple example.

Example. For $D = 3$, $D! = 6$ and $T = 256$. The permutations are

$$(321), (312), (231), (213), (132), (123),$$

$$\begin{aligned}
(10, 8, 9) \quad &\rightarrow \quad \pi_l = (3, 1, 2) \text{ type } 2,\\
(7, 3, 2) \quad &\rightarrow \quad \pi_l = (3, 2, 1) \text{ type } 1,\\
(5, 9, 6) \quad &\rightarrow \quad \pi_l = (1, 3, 2) \text{ type } 5.
\end{aligned}$$

Given a time series $\{x_i\}_{i=1}^{T}$, of length $T = 256$ an associated probability distribution is built using equation 2 above:

$$p(1) = \frac{\#\{k \mid k \leq T - D\,,\ I_k \text{ has type } 1\}}{T - D + 1},$$

where $T - D + 1 = 256 - 3 + 1 = 254$ in this case. In the same manner, $p(2), p(3), \ldots, p(6)$ can be calculated, corresponding to types $2, \ldots, 6$.

As it is discussed in [20], is important to remark that there are certain ambiguities related to the permutation entropy. We refer the reader to [2, 23, 24, 25] for further details on this quantifier. Here, we just stress the following points, which are relevant for our work:

- The permutation entropy can be considered as a measure of how unpredictable the behavior of a time series is. As such, it can be regarded as a measure of disorder of the time series.

- If the time series is strictly increasing (or strictly decreasing), then all its permutation patterns will be stored in a single symbolic pattern. Thus, the associated probability distribution will be maximally peaked, and consequently, its entropy will be the lowest. This is intuitively interpreted as follows: if a time series is monotonous, then its behavior is totally predictable and, as such, it has a low entropy. A similar consideration applies when the time series has long intervals with a definite monotonicity: the entropy will be close to the lowest value, since the contribution to the total entropy of those segments will be low.

- From the above point, it follows that the main contribution to the entropy comes from the segments of the time series for which the fluctuations are high. In this way, this quantifier allows to capture the intrinsic unpredictability of a given time series.

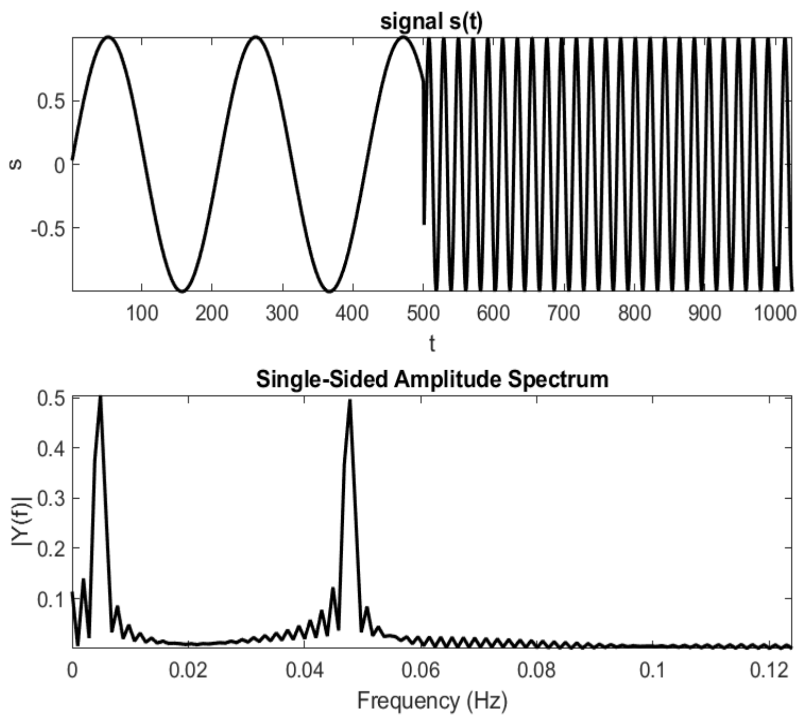

Figure 2: A test signal $s(t)$ and its Fourier spectrum.

- If the time series is constant in one of the intervals I_k (i.e., $x_k = x_{k+1} = \ldots = x_{k+D-1}$), then a convention must be taken in order to assign a per-

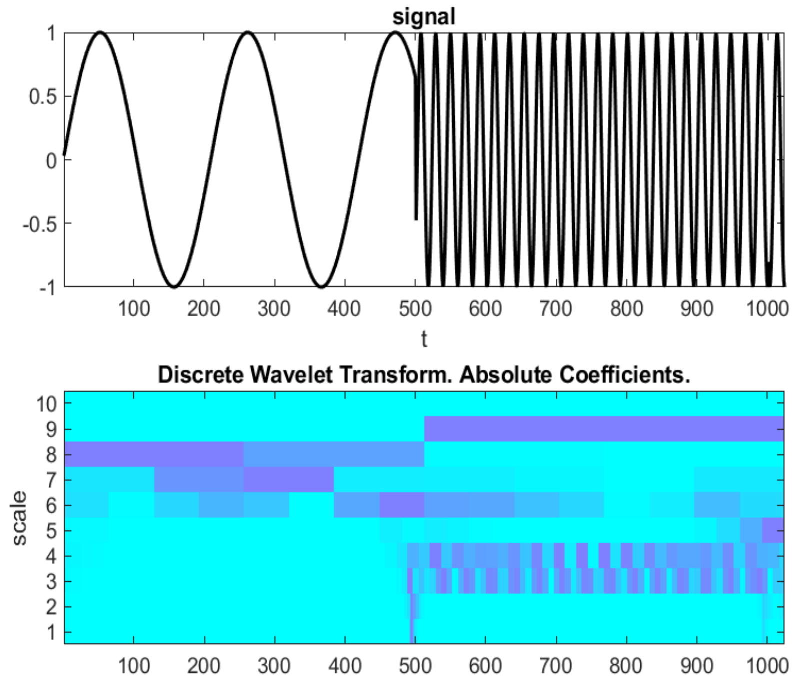

Figure 3: Discrete wavelet decomposition of the test signal.

mutation pattern to it. In this work, we adopt the convention $x_{k+i} = x_{k+i+1} \Longrightarrow x_{k+i} < x_{k+i+1}$. Thus, with this convention, if a time series is constant, its distribution will be maximally peaked, and consequently its entropy will be zero. This is convenient for us, given that, in the context of the COVID-19 pandemic, it is reasonable to interpret a constant sequence of data as a highly predictable time series. With our convention, if a country displays zero deaths during large periods of time, the permutation entropy will be low.

2.2 Wavelet entropy

Given a time series, the wavelet analysis performs a transformation that is sensitive to time and frequency. First, one fixes an orthogonal discrete wavelet basis [3, 17, 9, 4, 18]. By performing an adequate transformation, the time series is written as a linear combination in that basis. Its coefficients provide infor-

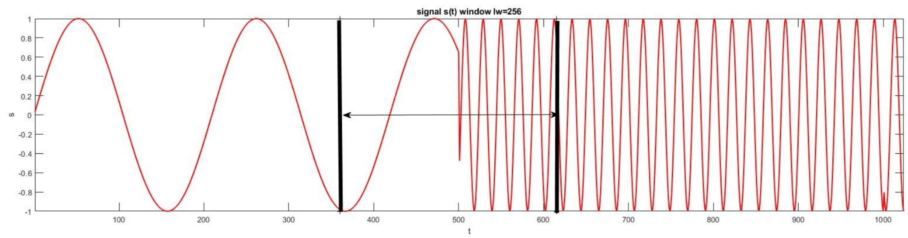

Figure 4: A test signal $s(t)$ and a window of length $lw = 256$.

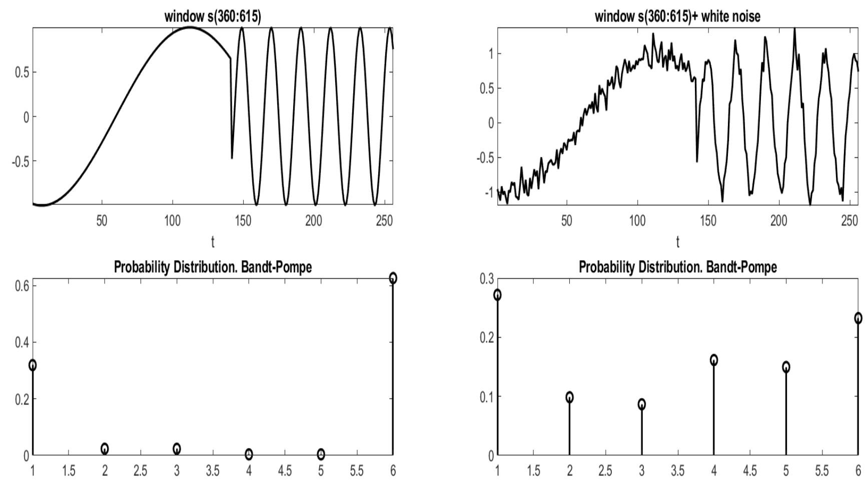

Figure 5: Permutation entropy of the window $S_I(p) = 0.49$ (left). Permutation entropy of the window adding white noise $S_I(p) = 0.96$ (right).

mation about the frequency spectrum associated with a localized time window. When normalized, a probability vector naturally arises, and its Shannon entropy is called the wavelet entropy. Given a signal $X(t)$ (assumed to be discrete), the wavelet expansion reads

$$X(t) = \sum_{j,k} C_j(k)\, \psi_{j,k}(t), \tag{4}$$

where $\{\psi_{j,k}(t),\ j,k \in Z\}$ is the wavelet family generated by translations and dilations of a given "mother wavelet", $\psi(t)$, which captures the detailed, high-frequency parts of the signal at each scale. Indices j and k correspond, respectively, to a dyadic frequency-decomposition and to time-translations [4].

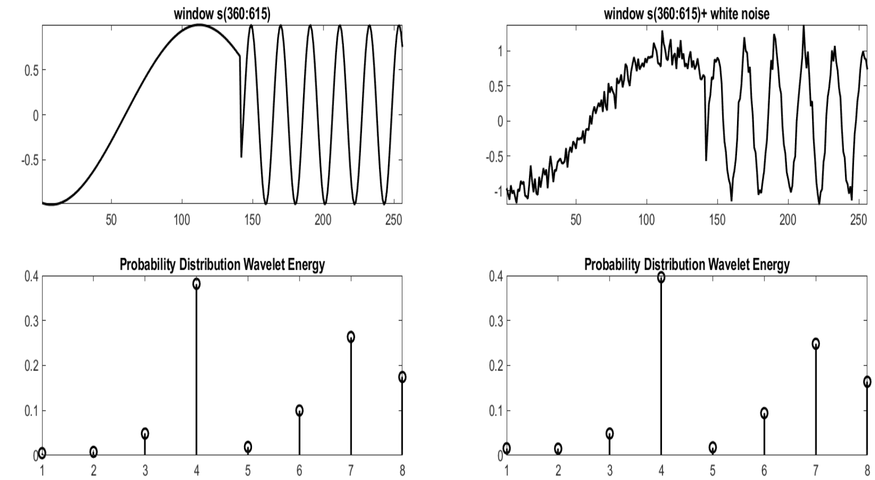

Figure 6: Wavelet entropy of the window $S_W(I) = 0.74$ (left). Wavelet entropy of the window adding white noise $S_W(I) = 0.75$ (right).

In this work we employ the orthogonal Daubechies-10 function (db10) as the mother wavelet.

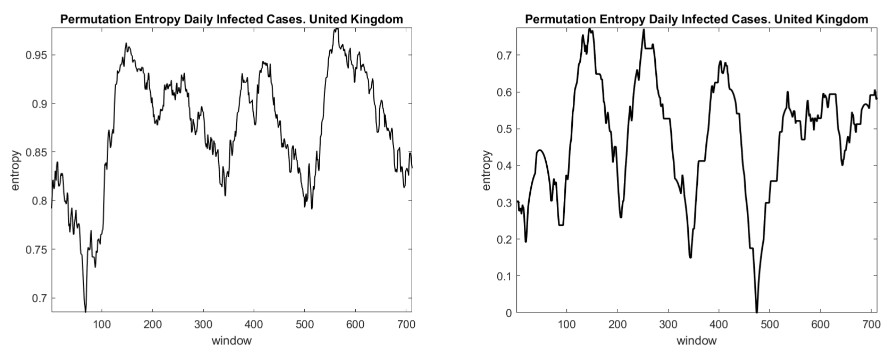

Figure 7: Permutation entropy with $D = 4$, using sliding window and moving average with $l = 1$ (left) and $l = 7$ (right) for United Kingdom.

A wavelet energy can be defined in similar way as in the Fourier theory. At each resolution level j, the wavelet energy of the signal is given by $E_j = \sum_k |C_j(k)|^2$. The wavelet coefficients correspond to the inner product between the signal and scaled and shifted versions of the mother wavelet, i.e. they are given by $C_j(k) = \langle X, \psi_{j,k} \rangle = \sum_{i=1}^{T} x_i \, \psi^*_{j,k}(t_i)$ for the discrete time

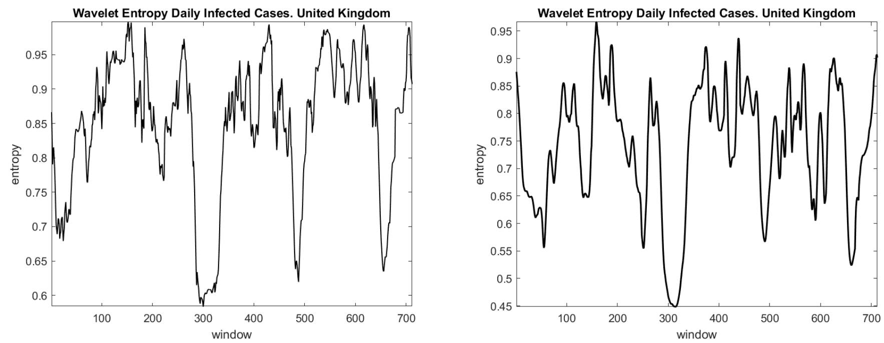

Figure 8: Wavelet entropy, using sliding window and moving average with $l = 1$ (left) and $l = 7$ (right) for United Kingdom.

series $\{x_i = X(t_i)\}_{i=1}^{T}$.

The total wavelet energy is obtained as

$$E_{tot} = \sum_j E_j = \sum_{j,k} |C_j(k)|^2 \, . \tag{5}$$

The relative wavelet energies for each resolution level,

$$p_j = \frac{E_j}{E_{tot}} \, , \tag{6}$$

constitute a set of normalized values which define the probability distribution of the wavelet energy in the time series. Clearly, $\sum_j p_j = 1$. The distribution $\{p_j\}$ can be considered as a time-scale density that constitutes a suitable tool for detecting and characterizing specific phenomena in both time and frequency planes.

2.3 Wavelet coherence

The continuous wavelet transform can be used to analyze time series that contain nonstationary power at many different frequencies [6]. The continuous wavelet transform of a discrete sequence x_n is defined as the convolution of x_n (with $n = 1, \ldots, N$ and uniform time intervals of size δt) with a scaled and translated version of a wavelet function ψ:

$$W^X(s,n) = \sum_{n'=0}^{N-1} x_{n'} \psi^{(*)} \left[\frac{(n'-n)\delta t}{s} \right] , \tag{7}$$

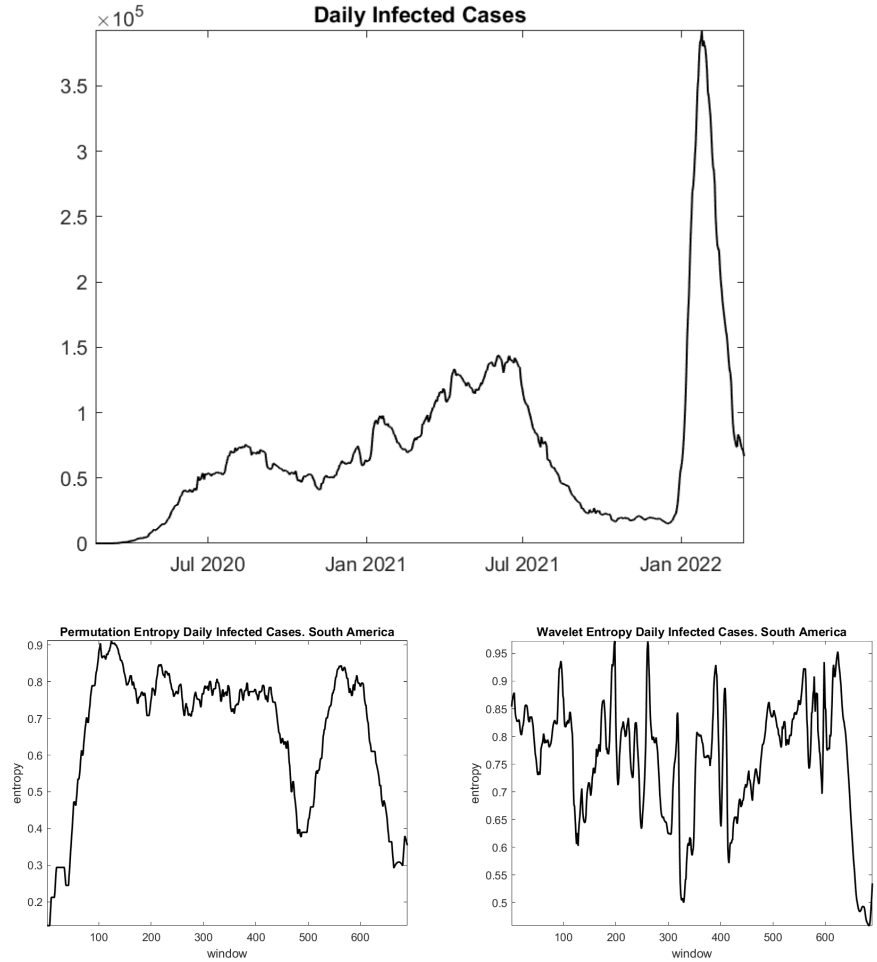

Figure 9: South America. Moving average (l=7). Daily Infected Cases (up). Sliding-window method: permutation entropy (down left) and wavelet entropy (down right).

where (*) indicates the complex conjugate. To be "admissible" as a wavelet, the function ψ must have zero mean and be localized in both time and frequency space. An example is the Morlet wavelet [27]. By varying the wavelet scale s and translating along the localized time index n, one can construct a picture showing both the amplitude of any features versus the scale and how this amplitude varies with time.

Wavelet analysis is also a useful tool for analyzing two time series together. The cross wavelet transform and wavelet coherence are presented for examining relationships in time–frequency space between two time series [27].

Given two time series x_t and y_t (with $t = 1, \ldots, N$ and uniform time intervals of size δt), the so-called cross wavelet transform of their respective (continuous) wavelet transforms $W^X$and W^Y is given by

$$W^{XY} = W^X (W^Y)^*, \tag{8}$$

and the cross wavelet power XWT is defined as $|W^{XY}|$. The phase of the complex number W^{XY} gives information about the phase relation between X and Y in the time-frequency space.

Another important measure –that we use in this work– is the wavelet coherence, which is given by

$$R^2(s,n) = \frac{|S(s^{-1}W^{XY}(s,n))|^2}{S(s^{-1}|W^X(s,n)|^2)S(s^{-1}|W^Y(s,n)|^2)}, \tag{9}$$

where S is a smoothing operator. The wavelet coherence provides a powerful tool for dealing with nonstationary time series.

3 Sliding-window analysis

As shown in a previous work [11], the informational content of the time series of the COVID-19 pandemic is low, in the following sense: when analyzed from the perspective of entropic quantifiers, the values obtained are high. One question naturally arises: why is the entropy so high? Under the light of the remarks of Section 2, it should be clear that the short and medium-term fluctuations in the random variables are responsible for the high values of the entropic quantifiers.

What is the nature of these short and medium-term fluctuations in the variables associated to the pandemic? It is clear that the methodology of data acquisition has a dynamics of its own. As an example, the report of the daily new cases will depend not only in the number of tests performed, but also in the machinery of data acquisition and publication by different agencies (such as governments). Also, the short-term behavior clearly depends on the eventual realization of social events and activities. Taken as a whole, these causes give place to a very complex dynamics in the short term. This behavior can be naturally characterized as "noise": researchers interested in the long-term behavior will discard it as much as possible. But it turns out that decisions must be taken, in certain circumstances, on a weekly basis. Thus, besides the long-term behavior, it is also desirable to try to understand better the complex short-term behavior. The sliding-window approach can be useful for characterizing the different stages of the evolution of the variables of interest, given that it considers the information associated to different time intervals. Here we focus in daily cases, but a similar analysis can be carried out for the daily deaths.

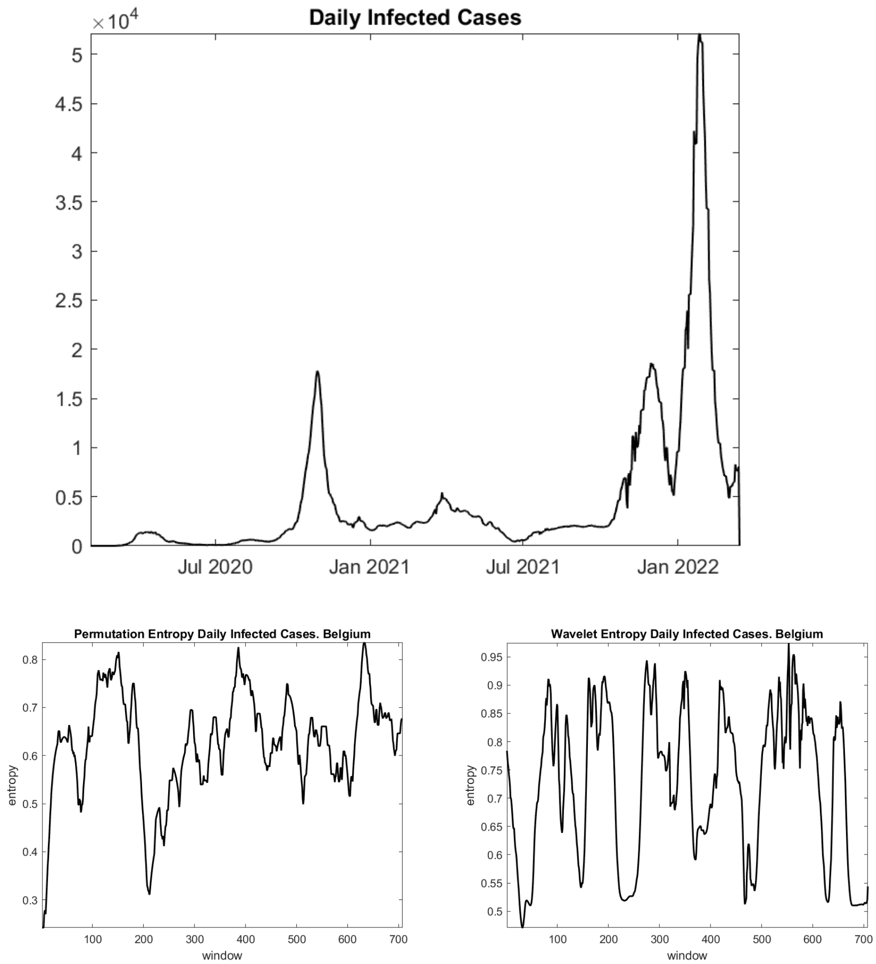

Figure 10: Belgium. Moving average (l=7). Daily infected cases (up). Sliding-window method: permutation entropy (down left) and wavelet entropy (down right).

One may wonder whether the entropy could be made smaller by getting rid of the fluctuations and smoothing the curves with a suitable method. This is important for studies that focus in the short-middle term behavior of the variables. A concrete possibility is to use a moving average, that replaces a given value of the random variable by the mean of an interval of length l centered at that point. As an example, the curves displayed in *Our World in Data* [22] use a moving average with $l = 7$. The resulting curve will depend on the chosen value of l. If l is high, we make the short-term fluctuations look smoother. But l cannot be taken too large (compared to the characteristic size of the variations of the curve), because doing so could destroy all the valuable information contained in the curve. Thus, which value for l should we choose? One possibility could be to use the value of l for which the entropy is minimal, while the large behavior of the curve is preserved. The reason for doing this is that entropy quantifies uncertainty: in order to maximize the information to be extracted from the data, one should thus minimize the entropy.

In order to see how the entropic quantifiers depend on the optimal value of the smoothing parameter l, let us illustrate with an example. In Figure 1 we show, as an example, the daily new cases of United Kingdom, first, without any smoothing (which is equivalent to $l = 1$), and then, the curve obtained for $l = 7$. Notice that the daily new cases display spikes not disappearing after application of the smoothing procedure.

3.1 Entropy quantifiers probability distributions for a test signal

Let us consider the test signal of Figure 2 which is plotted with its Fourier Spectrum. It has different frequencies at different times. In Figure 3 the Discrete Wavelet Transform absolute coefficients are shown.

In order to study the variation in time of the information-theory quantifiers a time window I_j of length lw is chosen, and the permutation and wavelet entropies, $S_P(I_j)$ and $S_W(I_j)$, associated with the interval I_j are calculated (see Figure 4).

In Figure 5, permutation entropy calculations are presented for the fixed window. The window considered of the test signal has intervals where it is strictly increasing or decreasing. As consequence the ordinal patterns are stored in two symbolic patterns, corresponding to that behavior, and the entropy has a low value. On the other hand, when white noise is added to the test signal (see Figure 5), fluctuations are high and the probability distribution is near the uniform distribution, with very high entropy.

The probability distributions obtained using wavelet entropy and the values of the entropy quantifiers are shown in Figure 6. In this case, the probability distributions are similar, they are not affected by the fluctuations.

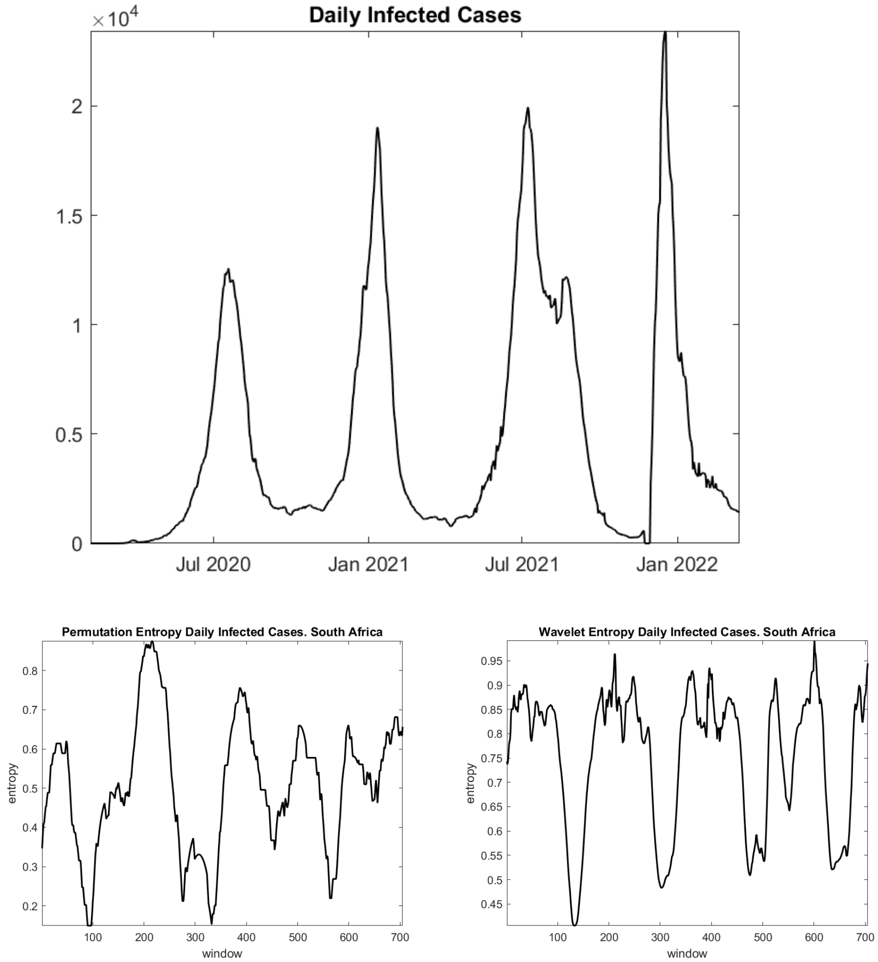

Figure 11: South Africa. Moving average (l=7). Daily Infected Cases (up). Sliding-window method: permutation entropy (down left) and wavelet entropy (down right).

3.2 Entropic quantifiers for time series of COVID-19

For the time series of the COVID-19 pandemic we use a sliding-window approach to study the change of the quantifiers in time. It consists in fixing a time window length w and computing the permutation and wavelet entropies, $S_P(I_j)$ and $S_W(I_j)$, associated with the intervals $I_j = \{x_j, x_{j+1}, \ldots, x_{j+w}\}$. Finally, we plot $S_P(I_j)$ and $S_W(I_j)$ as a function of j. This methodology is suitable for studying the behavior of a time series at different times [10].

As an example, in Figures 7 and 8 we show the permutations and wavelet entropies of the sliding windows, with $l = 1$, and using moving average ($l = 7$). As it was expected, the entropies decrease when the curve is smoothed.

In Figures 9, 10 and 11, we show the daily infected cases and the sliding-window curves for South America, Belgium and South Africa, respectively. For both entropies, windows of length $lw = 64$ were considered, and $D = 4$ was chosen in the permutation case.

With regards to the election of the relevant parameters in our computations, it is worth mentioning that the value of D must be consistently less than the number of points N of the analyzed data series. Bandt and Pompe have recommended to work with $D = 3, \dots, 7$. According to reference [23], $N \gg D!$ must be verified. Since we have taken $N = lw = 64$, the largest possible value of $D!$ so that at least $lw > D!$ is fulfilled, is 24 and thus we have chosen $D = 4$. We have also analyzed the data series with $D = 3$ and 5, obtaining conceptually similar results to those with $D = 4$. For the other parameter of the methodology, the embedding delay τ, there is a consensus in the literature to take $\tau = 1$, as originally considered by Bandt and Pompe [2]. We have also varied τ verifying consistency.

The mean values of the permutation and wavelet entropies associated with the sliding-window method for different countries, are depicted in Figures 12 and 13, for windows lengths = 64, and for $l = 1$ and $l = 7$. For those countries, we found that using $l = 7$ is suitable for minimizing the entropy. Notice that this methodology might depend on the given country, but also on the smoothing method chosen.

It is clear that the mean values tend to go down as the smoothing parameter grows. But not too much! Even when the empirical curves are smoothed (using moving averages), their informational content –from the point of view of information quantifiers– is still low. This result might be indicating that the dynamics of the different variables have an intrinsically complex dynamics (and that the complexity is not only originated in noise).

4 Wavelet coherence

In this section, we turn on the study of how the different time series are correlated. From the information-theory perspective, the wavelet coherence has been shown to be useful to study relations between time series, specially for nonstationary ones. Here, we formulate the following question: how do the curves for daily new cases and daily deaths correlate for a given country? One would expect that, after the successful application of public health policies

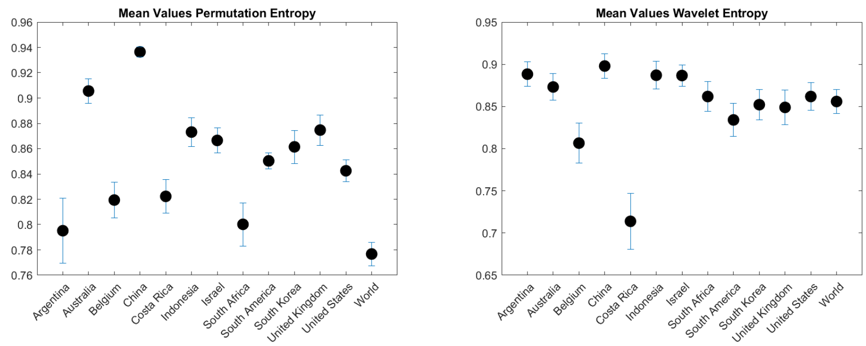

Figure 12: Mean values of permutation and wavelet entropies for different countries, $l = 1$.

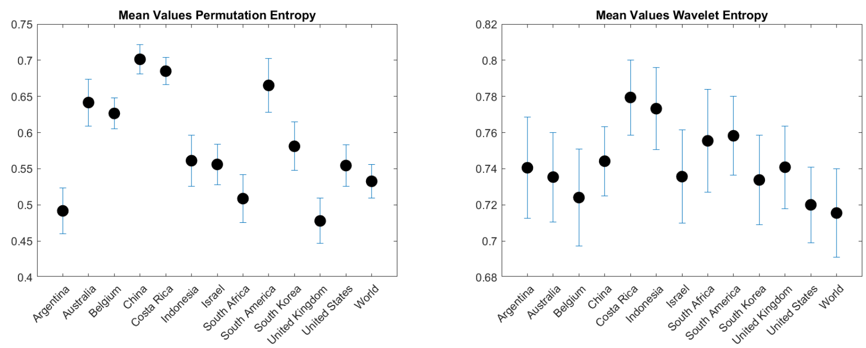

Figure 13: Mean values of permutation and wavelet entropies for different countries, with $l = 7$.

–such as vaccination campaigns and improvements in the health system–, the number of deaths drops off, even if daily new cases increase. How is it possible to quantify this? This is a nontrivial matter, given that different countries display different behaviors with regard to this point. The wavelet coherence has the advantage that it allows to compare different curves in the time–frequency plane. This allows for a powerful technique for detecting similarities between curves that can be nonstationary. We analyzed the wavelet coherence between daily new cases and daily deaths curves for different countries. One can check, for example, in Figures 14, that for Indonesia the daily new cases and daily deaths seem to show great degree of similarity in the time-frequency plane. This is reflected in the yellow zones (corresponding to high values, i.e. near 1) of the wavelet coherence. Contrarily, South America (see Figure 15) show a clear decoupling in the wavelet coherence, especially during the last months. This might reflect the effect of a successful health policies, possibly including vaccination campaigns. These results seem to indicate that the wavelet coherence could be considered as a suitable quantitative indicator for a change of behavior in the evolution of the pandemic.

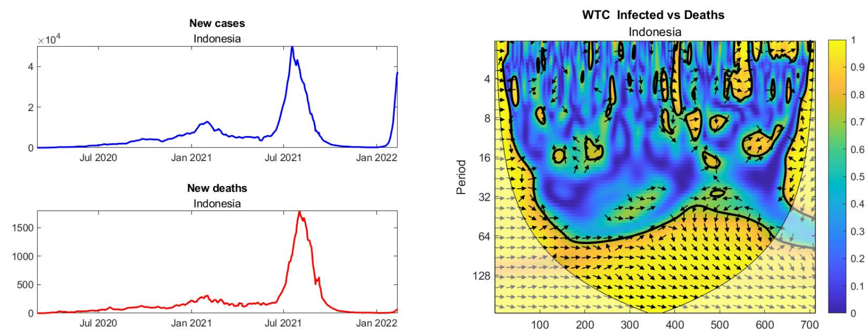

Figure 14: Indonesia. Wavelet coherence between daily new cases and daily deaths.

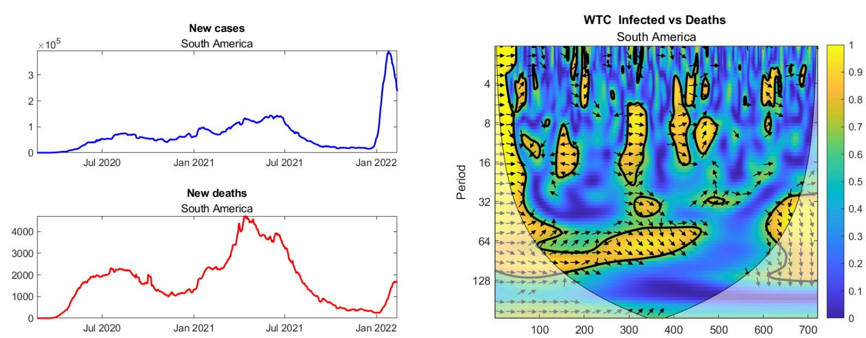

Figure 15: South America. Wavelet coherence between daily new cases and daily deaths.

5 Conclusions

In this work we have applied different information quantifiers for the study of the COVID-19 time series. Our contribution extends previous works and adds new aspects of the informational analysis.

We have analyzed the behavior of the sliding-window method for different degrees of smoothing. We have found that, even if the curves are smoothed, the mean values of the entropic quantifiers associated with the sliding-window method decrease, but can still be considered high. This results suggest a procedure for preprocessing data in mathematical modeling: if one wants to get rid of the highly unpredictable short and medium-term fluctuations, it is possible to choose the smoothing parameter in such a way that it minimizes the mean value of the entropy, while still preserving the general (long-term) shape of the curve.

In Section 4, we have studied the interdependence of the acquired data by appealing to the wavelet coherence. In particular, we have found that the daily new cases and daily deaths curves decouple in some countries. This can be attributed to the effective impact of public health policies including vaccination campaigns. The wavelet coherence seems to be a good candidate measure to quantify the change of behavior in the evolution of the pandemics. It could be expected in a further work a COVID-19 time series analysis using wavelet entropy in a two-dimensional context [18].

Taken as a whole, our results indicate that the analysis based on information quantifiers yields interesting clues to better understand the unpredictable behavior of the time series associated with the COVID-19 pandemic.

Acknowledgments and funding

ML, MP and FH acknowledge support from the National Research Council (CONICET), Argentina. AMK is supported by Comisión de Investigaciones Científicas de la Provincia de Buenos Aires (CICPBA), Argentina. Financial assistance from UNLP under the projects 11/I250 and 11/X812, and from CONICET under the project PIP 519, is also acknowledged. FH was partially funded by the project "Per un'estensione semantica della Logica Computazionale Quantistica- Impatto teorico e ricadute implementative", Regione Autonoma della Sardegna, (RAS: RASSR40341), L.R. 7/2017, annualità 2017- Fondo di Sviluppo e Coesione (FSC) 2014-2020.

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